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THE

PHYSICAL REVIEW.

NEUTRAL MAGNETIZATION IN IRON.

By Arthur W. Smith and Harry E. Hammond.

Synopsis.

Neutral Magnetization in Iron; Unsymmetrical Hysteresis Curves through the Zero Point; Comparison with the Normal Magnetization Curve.—Precise experimental measurements of unsymmetrical hysteresis loops passing through the zero point are given for rings of Swedish iron, cast iron and cast steel. Some writers have claimed that such a curve, after passing through the zero point, follows the normal magnetization curve. It is now shown that it does not follow the magnetization curve, but has a steeper slope at the zero point, and for some distance beyond, this slope depending upon the degree of magnetization at the tips of the corresponding hysteresis curve. Since the condition of the iron represented by the zero point on such a loop is not that of demagnetized iron, the authors call it *neutral magnetization*.

Normal Magnetization Curves: Method of Reversals and the Step-by-step Method.—The normal magnetization curves are also determined for these rings, both by the *method of reversals* and by the *step-by-step method*. Each method gives the same curve as nearly as can be shown in the figures.

Demagnetization of Iron; Allowable Frequency of Reversals; Eddy Currents.—These curves can be redetermined at any time, *provided that* the iron is first thoroughly *demagnetized*. This requires a sufficient interval of time between successive reversals of the magnetomotive force to allow the *eddy currents* in the iron to fall to zero. A special form of reversing rheostat is used to eliminate sparking and oscillations when the circuit is broken.

Conclusion.—The results show that iron, when brought to the zero point along any hysteresis curve, is in a definite state of magnetization, even though neutral. In this condition the iron has a greater permeability than when it is demagnetized to the zero point by slow reversals of a gradually decreasing magnetomotive force.

Introduction.

WHEN iron is subjected to an alternating magnetomotive force, it follows a definite hysteresis cycle after the first few reversals. This is usually represented by a hysteresis curve, each point of which represents a definite condition of the iron. Each point in the area enclosed by the hysteresis curve represents a possible condition which may be realized by a proper manipulation of the magnetic field intensity.

and many interesting forms of unsymmetrical hysteresis curves can be traced across this area.

This paper is concerned with one such point, viz., *O*, Fig. 1, which represents the condition of the iron when it is completely unmagnetized. The normal magnetization curve starts from this point, rising slowly at first, then more rapidly, and finally more and more slowly.

In explaining this double inflection of the magnetization curve Steinmetz[1] has pointed out that the same thing is also true of the lower branch of the hysteresis curve, and he concludes that "the rising magnetization curve determined by the step-by-step method is thus nothing but the rising branch of an unsymmetrical hysteresis cycle, traversed between such limits that the rising branch of the hysteresis cycle passes through the zero point." And curves are shown[2] where such unsymmetrical hysteresis curves run into, and are continued by, the rising magnetization curve "taken by ballistic galvanometer by the step-by-step method."

Similar curves are shown in text-books[3] in which the unsymmetrical hysteresis curve through the zero point does not agree with the rising magnetization curve as determined by the step by-step method. In neither case, however, is unmistakable evidence presented that these curves faithfully represent actual observation, rather than a free-hand sketch illustrating the opinion of the writer. It seemed worth while, therefore, to obtain curves from actual measurements which would show the true condition of the iron when brought to the zero point along such an unsymmetrical hysteresis curve. It is shown below that this condition is not that of demagnetized iron, but as there is no resultant magnetic flux it may be described as neutral magnetization.

OUTLINE.

The measurements of magnetic flux were made with the galvanometer-fluxmeter which has been previously used and described.[4] This instrument indicates the full amount of the change in the magnetic flux, even when the last part of this change is slow. The ordinary ballistic galvanometer shows only the change occurring in the first few seconds. Furthermore, the fluxmeter stands at rest after its deflection, and this facilitates an accurate reading. The scale was calibrated by comparison with a standard of mutual inductance, and the ratio of flux to deflection remained constant to within one tenth of one per cent. throughout the period of this investigation.

[1] Gen. Elec. Rev., Vol. XX., p. 135.

[2] Ibid., p. 138.

[3] *E.g.*, Watson, Physics, p. 716.

[4] PHYS. REV., Vol. 9, p. 415, 1917.

Three grades of iron have been investigated,—cast iron, cast steel, and Swedish iron. Each was in the form of a ring, and was wound uniformly with many turns of wire to carry the magnetizing current. A few turns of wire constituted the secondary circuit, which was connected directly to the galvanometer-fluxmeter. Four different B–H curves through the zero point have been determined for each of these rings, and are shown in the accompanying figures. Instead of long tables of data, the results of the many determinations are shown by the points on the curves.

DEMAGNETIZATION.

In view of the previous investigation[1] on the demagnetization of iron, each ring was fully demagnetized by many reversals of a slowly decreasing current before each set of measurements was made. With the Swedish iron ring it was necessary to reverse the current as slowly as once in ten seconds, when $H = 3$, or less, and to decrease it to a very small value, to obtain complete demagnetization. Anomalous results were obtained when this precaution was slighted.

It has been suggested that there may be oscillations in the primary circuit when the spark appears upon breaking the current. To avoid this possibility, a special rheostat was constructed by which the current could be reduced to zero by 128 steps, and increased as gradually, at each reversal. Thus the changes were slow enough to allow all eddy currents to die away, and it is certain that the iron was carried around the symmetrical hysteresis cycle corresponding to each cycle of the magnetizing field. The curves obtained after demagnetization by this method are the same as those obtained after demagnetization by simple reversal of the current, *provided*, that sufficient time is allowed between successive reversals.

NORMAL MAGNETIZATION CURVES.

The normal magnetization curve, OA, shown by the broken line in the figures, was obtained during the process of demagnetization. After a given value of the magnetizing current had been reversed ten times or more, the change in flux was measured when the current was reversed once again. Half of this change was taken as being the flux corresponding to this value of the current. The points thus determined are shown by the dots along the broken curves in Figs. 2, 3, and 4.

The same curve was also determined by the step-by-step method. Starting with the iron demagnetized, the current was suddenly increased by a definite amount. The corresponding change in the flux was meas-

[1] PHYS. REV., Vol. 10, p. 284, 1917.

ured by the fluxmeter, and then a second step was made. These points are shown by the small crosses along the broken line curve, *OA*. They agree so closely with the dots that one line represents both curves.

Both the dots and the crosses were plotted on millimeter cross section paper and a fine curve drawn through them. When this curve was retraced for Fig. 1, the broken line, *OA*, was wide enough to cover both sets of points. In order to show in Fig. 1, these points would have to be enlarged so much as to nullify their accuracy, and the result is better shown by the curve. They have been enlarged enough to show in the large scale Figs. 2, 3, and 4.

UNSYMMETRICAL HYSTERESIS CURVES.

The symmetrical hysteresis curve for Swedish iron is shown by *AP*, Fig. 1, for comparison with the unsymmetrical curves. This was ob-

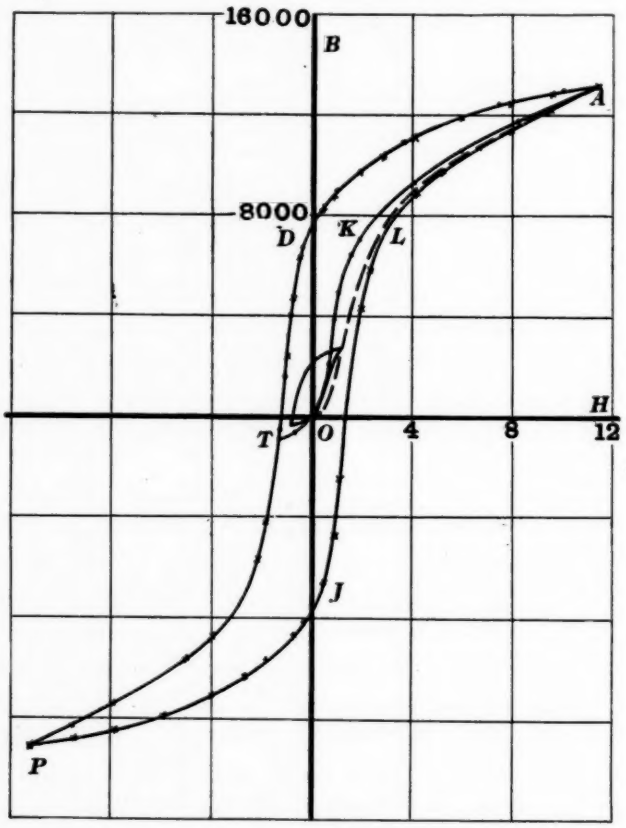

Fig. 1.

Hysteresis curve for Swedish iron, showing the relation of the unsymmetrical hysteresis loops through the zero point, *O*, to the full curve, *AP*. The larger loop is *ADTKA*, as shown by the dots.

tained by the direct deflection method[1] in which the location of each point is determined independently of the other points by measuring the difference between it and one of the tips of the curve. The points for this curve are shown by the crosses along AP. The dots along the curve show the unsymmetrical loop, $ADTKA$, which was traced in the same manner as the full curve, by measuring the distance of each point from the upper tip, A.

In order that the returning branch, TOK, shall pass through the zero point, O, the lower tip must be chosen at a definite point. If this is too far down the curve DTP, the returning branch of the unsymmetrical loop will pass below the zero point. If it is taken too high the returning branch will be above O. Whether it passes through O or not is determined by whether the change in flux along this curve when the field intensity is changed from zero to its full value is half of the change in

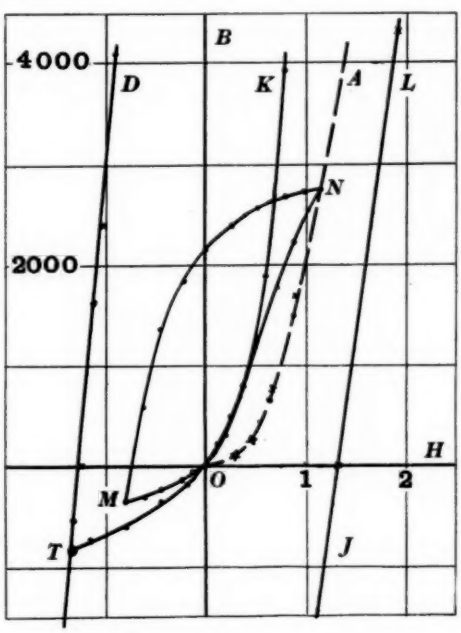

Fig. 2.

Curves for Swedish iron. A portion of Fig. 1, enlarged to show the direction of the unsymmetrical curves near the zero point. Both the smaller loop, MON, and the larger loop, TOK, are quite distinct from the magnetization curve, ONA.

flux for a reversal of the same field intensity. After several trials, a value was found for the reversed field intensity that would bring the iron along DT, and stop at T. From this point the returning branch passes through the zero point.

[1] A. W. Smith, Electrical Measurements, 3d imp., p. 181.

There is not the slightest indication that this returning branch of the unsymmetrical loop runs into the magnetization curve, for it passes through the zero point with a steeper gradient and it continues above the magnetization curve to the end at A. This is shown more clearly in Fig. 2, which is the central portion of Fig. 1 drawn to a larger scale.

The smaller unsymmetrical curve, NM, was determined by starting from a lower value of the field intensity, as shown at N, and carrying it to such a negative value at M that the returning branch of the curve would pass through O. The ordinate of each point on this loop is determined by measuring the change in flux between the point and N, while

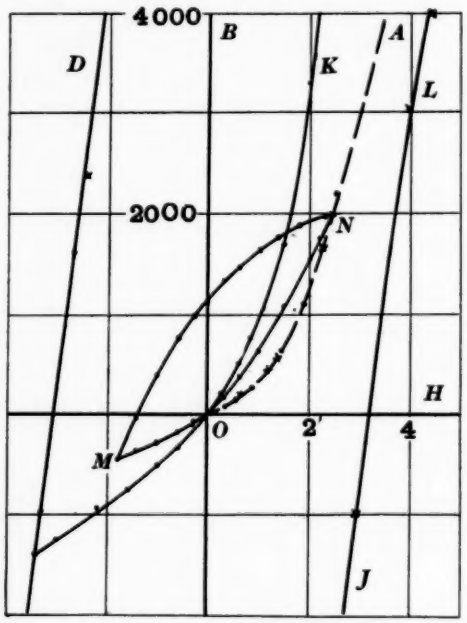

Fig. 3.

Curves for cast steel.

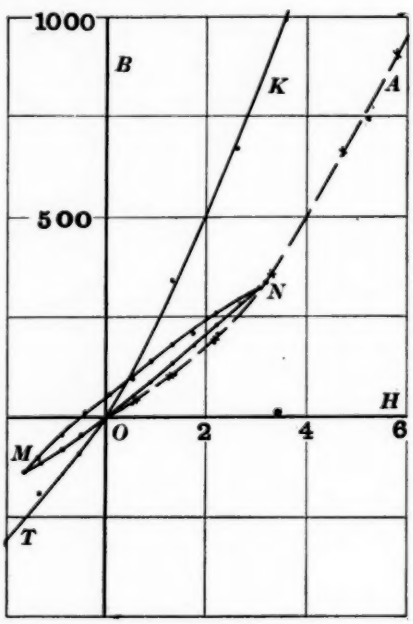

Fig. 4.

Curves for cast iron.

Unsymmetrical hysteresis curves through the zero point. *TOK* is the rising branch of a large loop. *MN* shows a smaller loop. The broken line, *ONA*, shows the normal magnetization curve, which does not coincide with either of the other curves.

the abscissa is given by the corresponding value of H as computed from the ammeter reading. This curve passes through O with a slope slightly smaller than TO, but it shows no tendency to run into the magnetization curve.

Fig. 3 shows a similar set of curves near the zero point for the case of cast steel. These show the same facts somewhat more clearly, as the curves do not run together as closely after leaving the zero point. Not only is the slope at O different for the three curves, but there is little in common throughout their entire lengths.

The curves for cast iron are shown in Fig. 4. On the scale to which these curves are drawn, the full hysteresis loop is wholly outside the limits of the figure. A portion of an unsymmetrical loop, *TK*, passing through the zero point is shown. Also the small loop, *MN*, which, even as small as it is, has a slope at *O* that is distinctly different from that of the magnetization curve, *ONA*.

CONCLUSION.

These curves show the manner in which iron, at the point of zero magnetization, responds to an increasing field intensity. When the iron has been brought along one of the unsymmetrical hysteresis loops passing through *O*, the magnetons are thereby somewhat organized, and the curve continues on through the zero point without running into the magnetization curve.

When, however, the iron has been demagnetized by many reversals of a decreasing field intensity, the magnetons are much more disorganized. This results in the low value of the permeability for weak field intensities, and gives the reverse bend at the beginning of the magnetization curve.

As the hysteresis loops, either symmetrical or unsymmetrical, are made smaller the iron is subjected to a smaller negative field intensity, and so is carried a smaller distance below the *OH* axis. The initial portion of the returning branch has the more gradual slope, and when the lower tip is near *O* this means that the whole branch is more flat than it is for the larger loops.

Only when the hysteresis loop has been made exceedingly small does its slope become that of the initial part of the magnetization curve.

UNIVERSITY OF MICHIGAN,
October, 1919.

CALCULATION OF THE CONSTANTS OF THE THREE-ELECTRODE THERMIONIC VACUUM TUBE.[1]

BY ROBERT W. KING.

SYNOPSIS.

Calculation of the Constants of the Three-Electrode Thermionic Vacuum Tube or Audion.—Given the characteristic equation of such a device as

$$I_p = \kappa\left(\frac{E_p}{\mu} + E_g\right)^{\eta}$$

it is shown that the value of the amplification constant μ is practically independent of the density of the space charge. It therefore follows that μ can be calculated in accordance with well-known principles in electrostatics from the dimensions of the tube. The calculation of μ as well of κ is carried out for four classes of tube structures.

1. The structure with plane-parallel elements in which the filament is symmetrically placed between grids and plates;

2. The structure with cylindrical anode and grid and a coaxial strand of filament;

3. The structure with cylindrical anode and grid and several parallel strands of filament arranged on an internal coaxial cylinder;

4. The structure similar to Class 3 except that the strands of filament lie on an external coaxial cylinder. These calculations result in the following formulas. *Class* 1. Given α the distance from plate to grid, β the distance from grid to filament, n the number of grid wires per unit length, r the radius of the grid wires, and A the total plate area,

$$\mu = \frac{2\pi\alpha n}{\log_e \frac{1}{2\pi r n}},$$

$$\kappa = 2.33 \times 10^{-6} \frac{A}{\sqrt{\alpha + \beta}} \left(\frac{\mu}{\alpha + \beta(\mu + 1)}\right)^{3/2}.$$

In this and the following values for κ it has been assumed that the exponent η has the value 3/2. This assumption in general gives good numerical agreement. *Class* 2. Given R_p, R_g, R_f the radii of plate, grid and filament respectively, and l the length of the structure,

$$\mu = \frac{2\pi R_g^2 \left(\frac{1}{R_g} - \frac{1}{R_p}\right) n}{\log_e \frac{1}{2\pi r n}}$$

$$\kappa = 14.65 \times 10^{-6} \frac{l R_p^{1/2} \mu^{3/2}}{[(R_p - R_g) + R_g(\mu + 1)]^{3/2}}.$$

Classes 3 *and* 4. The expression for μ is that given under Class 2. The expression for κ is

$$\kappa = 14.65 \times 10^{-6} \frac{l R_p \mu^{3/2}}{\sqrt{\pm (R_p - R_f)[\pm (R_p - R_g) \pm (R_g - R_f)(\mu + 1)]^{3/2}}},$$

where the signs are taken in such a way as to make all terms positive.

[1] The following paper was reported upon briefly at the meeting of the American Physical Society held in Philadelphia, October 11, 1919.

THE three-electrode vacuum tube or audion seems destined to play such an important part in the future development of electrical science, both as a laboratory instrument and as a device with ever-increasing industrial applications, that an attempt to establish theoretically the relations between its electrical constants and its structure and dimensions would seem appropriate.

Several years ago when the use of the audion in connection with telephone circuits was first contemplated by the engineers of the Bell System, it was realized that the satisfactoriness of the tube's operation depended to a large extent upon a proper choice of its constants. This led Dr. H. J. van der Bijl of this laboratory to conduct experiments with a view to finding empirical relations between tube dimensions and electrical properties. These relations have been of the greatest value in connection with the subsequent design of tubes for various purposes.

The following paper is an attempt to arrive at a similar set of relations by a theoretical analysis of the tube. The numerical agreement obtained in most cases is good, and is usually within the limits of variation found among a number of tubes of the same specifications.

As van der Bijl has shown[2] the characteristic of any vacuum tube is of the form

$$I_p = \kappa\left(\frac{E_p}{\mu} + E_g\right)^{\eta},$$

where E_p, E_g are the plate and grid voltages respectively and I_p is the space current. The constant μ is known as the amplification constant, and κ is related in a simple way to the internal resistance of the tube. The object of this paper is to deduce relations between these constants and the structure and dimension of any tube. The work is divided into two main divisions, the calculation of μ and κ for the type of structure possessing plane-parallel elements, and the application of these two results to the type with cylindrical elements. There are other tube constants such as input impedance and detection which are of considerable importance, but which lie without the scope of the present paper.

PART I. *a*.

Calculation of the Amplification Constant μ of a Vacuum Tube with Plane-Parallel Elements.

In attempting to relate the value of μ to the dimensions of a tube, the question of the effect of the "space charge" must first be disposed of. The constant μ measures the relative importance of plate and grid poten-

[1] The following paper was reported upon briefly at the meeting of the American Physical Society held in Philadelphia, October 3, 1919.

[2] PHYS. REV., 12, 180, 1918.

tials, and since the effect of the space charge will be to divert to itself some of the electric flux which would otherwise go to the filament, it seems reasonable to expect, provided this charge is largely confined to the space between grid and filament, that it will divert about equal fractions of the flux originating at the plate and grid, and thus leave μ unchanged. That this is essentially true is shown by the data given in Table I. These data were taken with a tube which showed a sharp "cut off" in the E_g, I_p characteristic, the μ being determined at various filament currents by measuring the value of E_g necessary to just reduce I_p to zero for two fixed values of E_p.

TABLE I.

Total Emission from Filament.	μ.
1,400 m.a.	15.0
240	15.2
47	15.6
10	15.8
.75	16.5

Since μ is practically independent of the presence of a space charge, it will be assumed that if ΔE_p and ΔE_g are increments of the plate and grid potentials which cause equal increments in the electric field at the surface of the cathode, the cathode being considered simply as an equi-

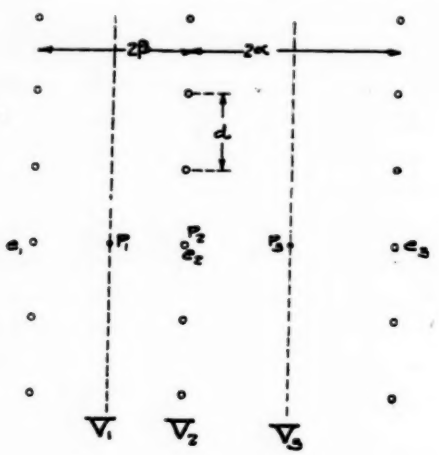

Fig. 1.

potential surface and not as a source of electrons, then μ will be the ratio $\Delta E_p/\Delta E_g$.

To calculate this ratio let us consider two infinite conducting parallel planes and a grid of parallel wires between them. We will suppose that the wires are of equal radius and that their radius is small compared

to the distance between adjacent wires or between the grid and either plane. Then the charge on each grid wire may be considered as concentrated along the axis of the wire. Let α and β be the respective distances from the planes to the center of any grid wire, d be the distance between the axes of adjacent wires, and r the radius of any wire. Also let V_1, V_2, V_3 be the potentials as shown in Fig. 1, and let e_1/d, e_2/d, e_3/d be the densities of surface charge, e_2 being the charge per unit length of grid wire. Now let three points P_1, P_2, P_3 be chosen as shown. For convenience they are taken to lie on a perpendicular which passes through the axis of a grid wire. Call $(V_1)_{e1}$ the potential at P_1 produced by the charge e_1, $(V_1)_{e2}$ the potential at P_1 produced by the charge e_2, etc.

Now let us replace the charges e_1 and e_3 by images of the grid in the two planes, thus obtaining the system of conductors shown in Fig. 1. Then since the potential at any point due to uniform distribution of electricity along a straight line of infinite length is

$$V_P = C_0 - 2e \log \rho,$$

where e is the charge per unit length and ρ is the distance from the line to the point P, we find, after summing over the infinite number of wires, and eliminating the constant C_0,

$$\begin{aligned} V_1 - V_2 &= \quad 2e_1A - 2e_3C + 2e_3D, \\ V_2 - V_3 &= -2e_1A + 2e_1B - 2e_3D, \end{aligned} \qquad (1)^1$$

where

$$A = \log_e \frac{\pi r}{d \sinh \frac{2\pi\beta}{d}},$$

$$B = \log_e \frac{\sinh \frac{\pi\alpha}{d}}{\sinh \frac{(\alpha + 2\beta)\pi}{d}},$$

$$C = \log_e \frac{\sinh \frac{\pi\beta}{d}}{\sinh \frac{(2\alpha + \beta)\pi}{d}},$$

$$D = \log_e \frac{\pi r}{d \sinh \frac{2\pi\alpha}{d}}.$$

[1] The equivalent of these equations is to be found in Maxwell's Electricity and Magnetism, Vol. 1, page 312, his derivation being based on the properties of conjugate functions.

Or if the ratios $\pi\alpha/d$, $\pi\beta/d$, etc., are greater than 2 we can write without appreciable error,

$$A = -\frac{2\pi\beta}{d} - \log_e \frac{d}{2\pi r},$$

$$B = -\frac{2\pi\beta}{d}, \tag{2}$$

$$C = -\frac{2\pi\alpha}{d},$$

$$D = -\frac{2\pi\alpha}{d} - \log_e \frac{d}{2\pi r}.$$

From equations (1) an expression for μ can readily be found. Suppose that plane (1) is the cathode of a vacuum tube and plane (3) is the anode. Then, solving equations (1) for e_1, we have, on the basis of the assumption that

$$\mu = \frac{\partial e_1}{\partial V_2} \Big/ \frac{\partial e_1}{\partial V_3},$$

which is evidently equivalent to the assumption that $\mu = \Delta E_p/\Delta E_g$,

$$\mu = \frac{C}{D - C};$$

or making use of the approximations in equations (2), which are probably sufficiently accurate for practical purposes,

$$\mu = \frac{2\pi\alpha}{d \log_e \frac{d}{2\pi r}}. \tag{3}$$

Letting n represent the number of grid wires per unit length equation (3) becomes

$$\mu = \frac{2\pi\alpha n}{\log_e \frac{1}{2\pi r n}}. \tag{4}$$

It is of interest to note that equation (4) gives μ as being independent of β, the distance between cathode and grid. The curves of Figs. 2 and 3, show the variation of μ with α and n as calculated by means of equation (4), and the plotted points represent experimental values.

PART I. *b*.

Calculation of the Internal Resistance of a Vacuum Tube with Plane-Parallel Elements.

The calculation of the internal resistance or the current corresponding to any given plate and grid potential, is somewhat complicated at the outset because of the variability of the exponent η in the equation

$$I_p = \kappa \left(\frac{E_p}{\mu} + E_g \right)^{\eta}. \qquad (5)$$

This exponent varies in a given tube with applied voltage, being usually as great as 2 when the effective voltage $(E_p/\mu) + E_g$ is comparable with the potential drop in the filament, and apparently approaching the

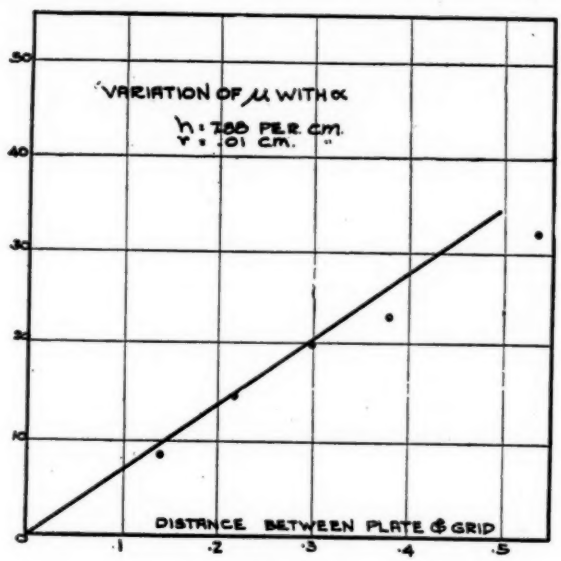

Fig. 2.

theoretical value 3/2 with increasing effective voltage. It also varies from tube to tube. Until the value of η can be foretold with considerable accuracy there is, therefore, little point to attempting great refinement in the theoretical derivation of κ of equation (5); but the following argument leads to a result which, without involving any arbitrary factor, gives a very good idea of the value of the space current which may be expected in any tube for any values of plate and grid voltage.

Imagine two structures each consisting of a plane anode and cathode (non-emitting) which are parallel and of indefinite extent, and let a grid be interposed between the anode and cathode of one structure. Suppose the distance between each anode and cathode to be $\alpha + \beta$ and the distance

between the grid and cathode to be β. Furthermore, let us suppose that the difference of potential E_p' when applied to the structure without the grid produces a certain charge density e on the cathode, and that the difference of potential E_p'' when applied between anode and cathode of the other structure (the grid being held at the same potential as the cathode) produces the same charge density e on that cathode. Since

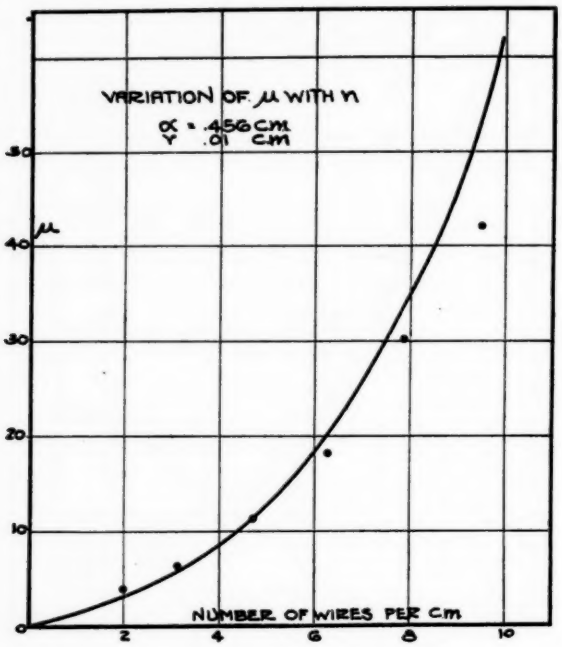

Fig. 3.

the charge densities on the two cathodes are equal, the fields at their surfaces will also be equal. Now the assumption will be made that the space currents per unit area which the potentials E_p' and E_p'' would produce, were the cathodes capable of indefinitely large emission, would be equal. In justification of this assumption, it may be pointed out that the space charge limits the space current by reducing the field just at the surface of the cathode to practically zero, and hence it is the value of the field very near the cathode rather than at remote points that is most influential in determining the space current. Obviously the potentials E_p' and E_p'' would, even in the presence of space charges, produce practically equal fields at the respective cathodes.

As Child has shown,[1] the current density in a plane-parallel structure

[1] C. D. Child, PHYS. REV., 32, 500, 1911.

without a grid is given by the equation

$$i = \frac{\sqrt{2}}{9\pi}\sqrt{\frac{e}{m}}\frac{E_p^{3/2}}{(\alpha+\beta)^2},$$

which becomes

$$i = 2.33 \times 10^{-6}\frac{E_p^{3/2}}{(\alpha+\beta)^2} \tag{6}$$

if the current is measured in amperes and potential in volts. Now, since the potentials E_p' and E_p'' correspond to equal surface charges on the cathodes, it follows from equation (1) that

$$\frac{E_p'}{E_p''} = \frac{\alpha+\beta}{\alpha+\beta(\mu+1)}. \tag{7}$$

Hence we find for the space current density in the structure with the grid,

$$I_p = 2.33 \times 10^{-6}\frac{1}{\sqrt{\alpha+\beta}}\left[\frac{E_p}{\alpha+\beta(\mu+1)}\right]^{3/2}. \tag{8}$$

This equation, though derived for a tube with a plane cathode, has been found to hold without change for the common type with filament cathode, provided the average distance between adjacent strands of filament is less than the distance $\alpha+\beta$. As the distance between adjacent strands is increased above the value $\alpha+\beta$, the measured value of the space current falls more and more below the value given by equation (8). The exact relation which this decrease follows has not as yet been determined. The effect is shown by the figures given for the Type V tube in Table II.

Combining equations (5) and (8) we have as the general expression for space current

$$I_p = 2.33 \times 10^{-6}\frac{A}{\sqrt{\alpha+\beta}}\left[\frac{E_p+\mu E_g}{\alpha+\beta(\mu+1)}\right]^{3/2} \tag{9}$$

and

$$\kappa = 2.33 \times 10^{-6}\frac{A\mu^{3/2}}{(\alpha+\beta)^{1/2}[\alpha+\beta(\mu+1)]^{3/2}},$$

in which A is the total area of the anode.

Since the space current in an audion is independent of the separation of the filament strands, within the limits noted above, it follows that it is also practically independent of the diameter of the filament.

It is interesting to note that the values of μ and I_p as given by equations (4) and (9) are independent of the unit of length used. Also it is evident that *any two audion structures, one of which is a large or small scale model of the other, have the same μ and current-voltage characteristics.* The truth of these propositions can readily be shown in other ways.

The extent to which computations based on equations (4) and (9) agree with experimental values is brought out in Table II.

TABLE II.

Average Type E Tube (*VT2* of the Signal Corps)

$\alpha = .466$
$\beta = .169$
$d_g = .290$
$r_g = .010$
$d_f = .476$
$L = 12.7$
$A = 12.0$

$\mu = \begin{cases} 6.6 \text{ calc'd.} \\ 6.1 \text{ obs'd.} \end{cases}$

$I_p(300,0) = \begin{cases} 79.5 \text{ m.a. cal'd.} \\ 69.0 \text{ m.a. obs'd.} \end{cases}$

Average Type O Tube.

$\alpha = .228$
$\beta = .169$
$d_g = .396$
$r_g = .010$
$d_f = .476$
$L = 12.7$
$A = 12.0$

$\mu = \begin{cases} 1.97 \text{ cal'd.} \\ 1.9 \text{ obs'd.} \end{cases}$

$I_p(120, -10.7) = \begin{cases} 70 \text{ m.a. cal'd.} \\ 64 \text{ m.a. obs'd.} \end{cases}$

Average Type V Tube.

$\alpha = .466$
$\beta = .169$
$d_g = .127$
$r_g = .010$
$d_f = .792$
$L = 5.30$
$A = 12.0$

$\mu = \begin{cases} 32 \text{ cal'd.} \\ 30 \text{ obs'd.} \end{cases}$

$I_p(100,0) = \begin{cases} 1.65 \text{ m.a. cal'd.} \\ 1.25 \text{ m.a. obs'd.} \end{cases}$

Tube No. 65345.

$\alpha = .317$
$\beta = .317$
$d_g = .159$
$r_g = .010$
$d_f = .635$
$L = 17.8$
$A = 22.6$

$\mu = \begin{cases} 13.6 \text{ cal'd.} \\ 13.0 \text{ obs'd.} \end{cases}$

$I_p(700,0) = \begin{cases} 112 \text{ m.a. cal'd.} \\ 99 \text{ m.a. obs'd.} \end{cases}$

PART II. *a*.

Constants of the Cylindrical Type of Tube.

Tubes of the cylindrical type fall into three classes:

1. Those with cylindrical anode and a single strand of filament along the axis.
2. Those with several parallel strands of filament arranged on the surface of a cylinder within and concentric with the anode, and whose radius is comparable with that of the anode.
3. Those similar to class 2 except that the filament is outside of the anode.

The results obtained in Part I. *a* can readily be extended to yield

about as accurate values for cylindrical tubes. This may be accomplished by use of the transformation known as inversion with respect to a circle. Under inversion both charge and potential remain invariant.

Consider a vacuum tube structure with plane-parallel elements of infinite extent. Let Fig. 4 represent a section of the structure, and imagine this section to be inverted with respect to a circle as shown. After inversion the tube elements are arranged around circles which lie tangent to each other at the center of inversion. Now since both charge and potential have remained the same, it follows that these circles must represent a cylindrical structure with the same constants as the plane-parallel structure. In particular the section *aa* must have the same constants as the section *AA*. But the error involved in taking *aa* to be a section of a structure with its center at the center of inversion is likely to be small. That is, the plane-parallel structure and the cylindrical structure shown by the dotted circles in Fig. 4 will have approximately the same constants. Hence, calling R_f, R_g, R_p the radii of cathode circle, grid circle and anode respectively, R the radius of the circle of inversion, n_g, ν_g, the number of grid wires per unit length in the cylindrical structure and in the plane structure formed by inversion respectively, r_g, ρ_g, the radii of the grid wires respectively, and α the distance between anode and grid in the plane structure, we have by equation (4),

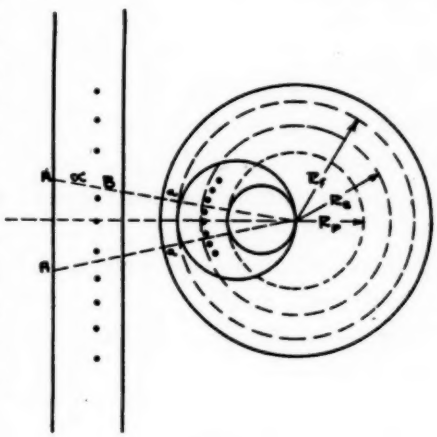

Fig. 4.

$$\mu = \frac{2\pi\alpha\nu_g}{\log_e \frac{1}{2\pi\nu_g\rho_g}}, \tag{10}$$

in which

$$\pm\alpha = R^2\left(\frac{1}{R_p} - \frac{1}{R_g}\right),$$

$$\nu_g = \frac{R_g^2}{R^2} n_g, \tag{11}$$

$$\rho_g = \frac{R^2}{R_g^2} r_g;$$

or combining equations (10) and (11)

$$\mu = \frac{\pm 2\pi n_g \frac{R_g}{R_p}(R_p - R_g)}{\log_e \frac{1}{2\pi n_g r_g}}. \qquad (12)$$

It is evident that the minus sign is to be used when dealing with tubes of class 3.

This argument fits particularly the type of tube in which the grid wires are parallel to the axis; but it seems that for any given spacings of elements and grid wires, it makes little difference whether the grid wires run parallel or perpendicular to the tube axis. Obviously the more nearly equal R_f, R_p, R_g are and the closer the grid mesh, the more accurate the above result will be and it might very reasonably be expected to fail badly in the case of a tube of class 1. Such, however, does not seem to be the case, probably because μ is practically independent of the distance β.

PART II. *b*.

We will now note a certain relationship which exists between plane-parallel and cylindrical structures which contain no grids. In Fig. 5 there is represented a section of a cylindrical structure the radius of whose anode, R_p, is large compared with the radius of the cathode R_f (for our present needs R_p will be considered at least ten times as large as R_f). In Fig. 6 there is also shown a section of a plane-parallel structure, the distance between the elements being R_p and their width $2\pi R_p$. Now we know that the space current per unit length (taken perpendicular to the paper) in the plane-parallel structure is

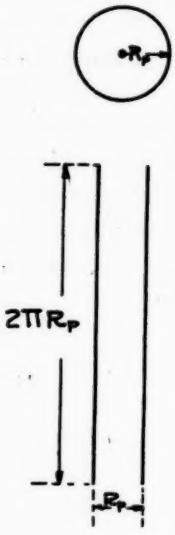

Fig. 5.

$$i = 2.33 \times 10^{-6} \frac{2\pi R_p V^{3/2}}{R_p^2},$$

$$= 14.65 \times 10^{-6} \frac{V^{3/2}}{R_p}.$$

But this is also the space current per unit length in the cylindrical structure.[1] Hence the two structures will give equal space currents.

Now imagine a grid added to each structure in such a way that each grid is at the distance R_g from its cathode and that each structure has the same μ. Then they will again have practically equal space currents,

[1] See Langmuir, PHYS. REV., 2, 459, 1913.

and it follows from equation (8) that for the cylindrical structure of

$$\textit{Class 1,}\quad I_p = 14.65 \times 10^{-6} \frac{l}{R_p}\left[\frac{(R_p - R_f)(E_p + \mu E_g)}{(R_p - R_g) + (R_g - R_f)(\mu + 1)}\right]^{3/2},$$

in which l is the length of the structure; or, since R_f is practically negligible in comparison with R_p and R_g,

$$I_p = 14.65 \times 10^{-6}\, l\sqrt{R_p}\left[\frac{E_p + \mu E_g}{(R_p - R_g) + R_g(\mu + 1)}\right]^{3/2}. \quad (13)$$

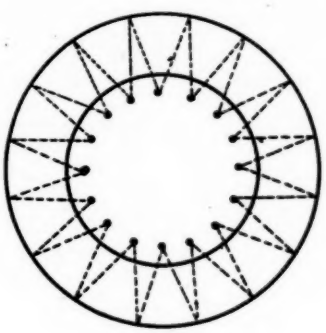

Fig. 6.

Similar expressions for Classes 2 and 3 can be found by considering tubes of these types to be built up of sections of Class 1 as shown in Fig. 6. On this basis we obtain the following equations:

Class 2,

$$I_p = 14.65 \times 10^{-6} \frac{R_p l}{\sqrt{R_p - R_f}}\left[\frac{E_p + \mu E_g}{(R_p - R_g) + (R_g - R_f)(\mu + 1)}\right]^{3/2}, \quad (14)$$

Class 3,

$$I_p = 14.65 \times 10^{-6} \frac{R_p l}{\sqrt{R_f - R_p}}\left[\frac{E_p + \mu E_g}{(R_g - R_p) + (R_f - R_g)(\mu + 1)}\right]^{3/2}, \quad (15)$$

where l is again the length of the structure. These equations are independent of the number of strands of filament and may be expected to hold closely only when, upon development as indicated in Fig. 6, the distance between adjacent strands is found to be less than the distance between anode and cathode circles.

Table III. shows the numerical agreement between the above expressions for μ and I_p in several different tubes. All dimensions are again in centimeters and the currents in milliamperes. The number of strands of filament (which, in the tubes given below, were parallel and equally spaced except where noted) is represented by N.

TABLE III.

Class 1, *Tube No.* 1329. (French Transmitting Tube).

$R_f = .00635$
$R_g = .2$
$R_p = .475$
$d_g = .155$
$r_g = .0125$
$l = 1.5$
$N = 1$

$\mu = \begin{cases} 6.9 \text{ cal'd.} \\ 7.4 \text{ obs'd.} \end{cases}$

$I_p(100,0) = \begin{cases} 6.1 \text{ m.a. cal'd.} \\ 4.4 \text{ m.a. obs'd.} \end{cases}$

Class 2, *Tube No.* 48128.

$R_f = 1.03$
$R_g = 1.19$
$R_p = 1.59$
$d_g = .159$
$r_g = 0.127$
$l = 3.18$
$N = 6$

$\mu = \begin{cases} 17 \text{ cal'd.} \\ 20 \text{ obs'd.} \end{cases}$

$I_p(1100,0) = \begin{cases} 610 \text{ m.a. cal'd.}^1 \\ 120 \text{ m.a. obs'd.} \end{cases}$

Class 2 *or* 3, *Tube No.* 18380.

$R_f = 1.35$
$R_g = .783$
$R_p = .635$
$d_g = .371$
$r_g = .019$
$l = 12.7$
$N = 15.$

$\mu = \begin{cases} 2.3 \text{ cal'd.} \\ 3.3 \text{ obs'd.} \end{cases}$

$I_p(700,0) = \begin{cases} 917 \text{ m.a. cal'd.}^2 \\ 613 \text{ m.a. obs'd.} \end{cases}$

Class 3, *Tube No.* 67962.

$R_f = .85$
$T_g = .635$
$R_p = .508$
$d_g = .254$
$r_g = .33$
$l = 10$
$N = 16$

$\mu = \begin{cases} 8.6 \text{ cal'd.} \\ 8.9 \text{ obs'd.} \end{cases}$

$I_p = (750,0) = \begin{cases} 795 \text{ m.a. cal'd.} \\ 750 \text{ m. a. obs'd.} \end{cases}$

In conclusion the author wishes to acknowledge indebtedness to Dr. H. J. van der Bijl for permission to use many of his experimental results.

RESEARCH LABORATORIES OF THE
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WESTERN ELECTRIC COMPANY, INC.,
NEW YORK CITY.

[1] In this tube the filament strands were grranged in pairs, so that the effect was practically that of three strands. If a diagram of the tube is made it will be seen that in order to make the assumption true on which the calculation of equation (14) is based, the tube should have about fifteen strands of filament. If, therefore, we divide the value of I_p given above by five, we obtain 122 m.a.

[2] Using the observed values of μ in place of the calculated in computing I_p gives $I_p = 640$ m.a.

A PRINCIPLE OF DUALITY IN THERMODYNAMICS.

BY ARTHUR C. LUNN.

SYNOPSIS.

Statement of the Principle and Comparison with Dualities already Known.—It is shown that the general relations in thermodynamics can be arranged so as to exhibit a duality such that energy is dual to entropy and temperature to its reciprocal, leading to a systematic scheme of transformation of variables which may have value similar to the analogous principles already recognized in geometry, mechanics, and electromagnetic theory. It is pointed out that each physical concept has thus two corresponding modes of analytic representation and the same analytic form two physical interpretations.

Applications.—The working of the principle is illustrated by the interrelations of physical coefficients belonging to a fluid, and an example of symmetry of notation for entropy and energy equations and other pairs of formulas; by the transformation of thermodynamic potentials; and by the dualistic relation of the two Gibbs criteria of equilibrium and stability.

THE principle of duality in geometry has been long recognized as of fundamental importance, and by virtue of suitable definitions and the introduction of certain ideal elements has been rendered perfect, or complete, in the sense that no exceptions to its applicability occur when the various elements involved are properly interpreted. Certain analogues of this principle occur in physical science, notably the well-known duality of translation and rotation and the duality exhibited by Heaviside in the theory of the electromagnetic field. There are also in the periodic scheme of the chemical elements indications of something similar, which may possibly prove to be interpretable in terms of the duality of geometry, for instance in connection with the cubic model for valences.

These physical dualities, in contrast with that of the ideal geometry, prove to be in some ways incomplete when compared with the details of experimental fact, but this can hardly be considered to reduce their value. It may be said that both the range of their validity and their features of incompleteness are matters of physical significance.

Thus the translation of a particle and the rotation of a rigid body exhibit corresponding or mutually dual features; for example, velocity, momentum, and force in the one, and angular velocity, angular momentum and torque in the other. But the mass of a particle, even if distinguished into longitudinal and transverse, is somewhat more special

in character than the set of moments of inertia of a rigid body; and in contrast with velocity and momentum the vectors of angular velocity and angular momentum do not in general have the same direction.

Heaviside's duality not only furnishes a simplifying principle for use in the formulation of the theory but suggests in connection with any experimental situation the search for its dual, which is sometimes relatively inaccessible to direct study merely because of the wide difference in the orders of magnitude involved. But the absence of magnetic conduction current, the solenoidal character of the magnetic induction, the fact that electric susceptibility is always positive, are also matters of concern to a theory of electromagnetism, and are actually subject to at least partial explanation by the electron theory.

The duality of geometric theory has already found application in thermodynamics. It has been pointed out by Bryant[1] that the (v, η, ϵ) and (p, t, ζ) surfaces, in the Gibbs notation, are dual in the sense of polar reciprocation with respect to a certain paraboloid of revolution, and like relations are suggested with respect to other pairs of surfaces, especially those related to the thermodynamic potentials. A similar transformation exists involving the duality of the mass of any constituent and its chemical potential in any single phase of a complex. In the Gibbs theory it appears that pressure, expressible in terms of temperature and concentrations only, can act as characteristic function for the constitution of the phase, apart from total mass. This fact is known to be connected with the homogeneity of the energy as a function of the masses, and this can be thought of as represented by a corresponding degeneration of a quadric manifold of reference.

The general transformation implied in these cases, of which Bryan's example gives the type, is the interchange of any coördinate with the corresponding generalized force. It corresponds therefore to the Legendre transformation in the theory of the canonical equations of dynamics, which was foreshadowed by Hamilton in setting up his "third function." It may be viewed as exhibiting a general type of duality in dynamics, which is extensible to thermodynamics by making temperature dual to entropy and any energy function dual to some other energy function, as illustrated in the interrelations of the familiar thermodynamic potentials. The duality to be considered in the present paper is of a different type, and seems to be in a way intrinsic to the thermodynamics itself, apart from methods of geometric representation or dynamical analogies.

One can hardly avoid noticing the parallel between energy and entropy

[1] Bryan: Thermodynamics, 175–176.

in the Gibbs criteria of equilibrium, or between functions having these respective physical characters when used as characteristic functions. Moreover, the fact that the same quantity of heat may be said to exhibit both energy and entropy, as distinct measurable properties to some extent on a parity,[1] also suggests the possibility of an arrangement of ideas such that a correspondence of these two concepts would occur as one item in a full-fledged duality, where then the dual of temperature would be its reciprocal. The following arrangement shows that such a duality does exist and so far as concerns the general formulas is complete, although as in the other cases mentioned there are significant differences in detail between a given physical concept or relation and its dual. There could be a certain flexibility as to algebraic signs in the definitions, the choice given being apparently on the whole the most convenient.

Consider first reversible processes, and let mere differential expressions be distinguished from true differentials by the symbols δ, d respectively. Let δq be thought of as the energy measure of the entropy communicated to the system, and define as its dual δe, the entropy measure of the energy given out. As dual to δw, the energy measure of the work done by the system, define δr, the entropy measure of that work, or more exactly the entropy measure of heat equivalent in energy value to that work. Then the fundamental relations are

$$(1) \qquad \delta q = t ds, \quad \delta q - \delta w = du, \quad \delta r = \delta w/t, \quad \delta e = (\delta w - \delta q)/t,$$

transcribing the two laws and the two definitions. If the latter be solved for δe and δr in terms of δq and δw, and the values be substituted in the equations for du and ds, there results with change of order the equivalent system of equations

$$(2) \qquad \delta e = -du/t, \quad \delta e - \delta r = -ds, \quad \delta w = t\delta r, \quad \delta q = (\delta r - \delta e)t.$$

It is at once seen that a complete symmetry or duality is present, corresponding to interchange of δq with δe, δw with δr, u with $-s$, and t with $1/t$.

An isothermal or an adynamic process is thus self-dual; the dual of an isentropic process is what is often though unfortunately called isodynamic, or of constant internal energy. Of the six cycles corresponding to pairs chosen from the four quantities u, s, t, w, the (t, w) cycle of Rankine and Stirling and the (u, s) cycle are self-dual. The Carnot cycle (s, t) has for dual the isodynamic-isothermal or (u, t) cycle, while the (u, w) and (s, w) cycles correspond. The dual of the free energy $\psi = u - st$ is $-s + u/t = \psi/t$; similarly the duals of the other Gibbs

[1] See paper by the writer, PHYS. REV., N.S., 14, 1–19, 1919, and the references there cited.

potentials $\chi = u + pv$ and $\zeta = \psi + pv$ will be shown to be $(pv - st)/t$ and ζ/t respectively. Thus the transformation of duality exhibits in a systematic way the correspondence between formulas involving energy potentials and similar formulas involving what may be called entropy potentials, such as have been used by Massieu, Planck, and others.

The general interchange described could be accompanied by an arbitrary transformation of the configuration coördinates; for the simplest application of the principle, however, each configuration coördinate may be regarded as its own dual. It will be noticed that if δq, δw, t be considered as determined by observation, then all other quantities in the general relations (1) and (2) can be computed from them without reference to the specific properties of the individual system involved; in particular δe and δr may be regarded as in this sense also subject to experimental determination. Those relations are thus to be considered as general for all systems; when exhibited in terms of the properties of a specific system they determine the corresponding special form of the general duality.

General Transformation of Coefficients.

If such a system be described in terms of t and the configuration variables $v_1 \cdots v_n$, the latter taken as self-dual, the alternative forms are

$$\delta q = k dt + \Sigma l_i dv_i, \qquad \delta w = g dt + \Sigma p_i dv_i; \tag{3}$$

$$\delta e = \kappa d\tau + \Sigma \lambda_i dv_i, \qquad \delta r = \gamma d\tau + \Sigma \pi_i dv_i; \tag{4}$$

where

$$\tau = 1/t, \quad \kappa = (k - g)/t, \quad \lambda_i = (p_i - l_i)/t, \quad \gamma = g/t, \quad \pi_i = p_i/t. \tag{5}$$

If t is an "internal" variable g and γ will vanish; if t is not taken as one of the independent variables both g, k and their duals will be zero. Convenience of reckoning may sometimes be improved by use of a special set of units, for example in connection with critical point phenomena by making $t_c = 1$; or with arbitrary units the transformation of temperature could be $\tau = t_c^2/t$, with corresponding changes in the other formulas.

If then any physical concept or relation be describable in terms of the variables and coefficients in (3) there will correspond to it a dual concept or relation describable in a similar way in terms of the corresponding coefficients in (4), and conversely. Also the same concept or relation can be described in two ways according to which set of equations or coefficients is made fundamental. Thus there are dual physical interpretations of the same analytic form and two analytic forms having the

same physical interpretation. The complete theory based on (4) is of course equivalent to that based on (3), but there may be a certain gain in symmetry of arrangement or convenience of choice of variables secured through the recognition of the presence of this duality. Indeed, certain correlations of physical properties, such as those pointed out by Trouton and Eötvös, find convenient expression in terms of entropy rather than energy.

The two general laws of thermodynamics demand that $\delta q - \delta w$ and $\delta q/t$ shall be exact differentials du and ds; the dual conditions, equivalent to the originals interchanged, are that $\delta e - \delta r$ and $\delta e/\tau$ shall be exact differentials, these being $-ds$ and $-du$ respectively. The Clapeyron and Clausius conditions

$$l_i = t\frac{\partial p_i}{\partial t}, \qquad \frac{\partial k}{\partial v_i} = t\frac{\partial^2 p_i}{\partial t^2},$$

which apply when g vanishes, have for duals

$$\frac{p-l}{t} = -t\frac{\partial}{\partial t}\left(\frac{p}{t}\right), \qquad \frac{\partial}{\partial v}(tk) = -t\frac{\partial}{\partial t}\left[-t^2\frac{\partial}{\partial t}\left(\frac{p}{t}\right)\right],$$

which are separately equivalent to their originals.

FLUIDS.

As type of a simple system in normal form consider the case of a fluid, characterized by the values of k, l, p, as functions of v, t, where v denotes volume so that $g = 0$. All coefficients determinable in terms of these, like coefficients of elasticity and expansion, have their theoretical duals, some of which have simple interpretations. The dual of l, the latent heat of expansion, is $\lambda = (p - l)/t$, the negative of the internal latent entropy of expansion. The vanishing of λ is one property of an ideal gas, whose energy is a function of temperature only; similarly the vanishing of l is a property of the ideal solid as sometimes defined, whose entropy is a function of temperature only. The duals of the four Maxwell relations like their originals are all equivalent to the Clapeyron relation.

The dual of the isochoric derivative $\partial p/\partial t$ is $\partial\pi/\partial\tau = p - l$, the negative of the internal latent heat of expansion. The interpretation of $\pi = p/t$, dual to p, depends on the nature of the particular fluid considered; for an ideal gas π is proportional to the volume, making the dual of an isobaric process in this case isochoric. In general π/R may be thought of as a kind of reduced or fictitious volume for comparison with an ideal gas at the same temperature and pressure.

For some purposes the dual variables may be more convenient than the original, as in equations of state for gases. The departures of an

actual gas from the ideal relations can be exhibited by contour lines in the neighborhood of the single hyperbolic curve $\pi v = R$. To match this the dual of the Clapeyron formula indicates values of λ in the neighborhood of zero, in lieu of values of l in the neighborhood of the variable p.

An instance of the symmetry of notation which is sometimes possible on account of this reciprocity is given by the case of a fluid such that the free energy and its dual have the forms

$$\psi = f(v) + t\varphi(v) + a(t), \tag{6}$$
$$\psi/t = \varphi(v) + \tau f(v) + \alpha(\tau),$$

where $f(v)$ and $\varphi(v)$ are functions independently arbitrary in analytic form, though dual to each other in position, while $a(t)$ and $\alpha(\tau)$ are related by

$$\alpha(\tau) = a(t)/t. \tag{7}$$

Then it follows that

$$u = f(v) + a(t) - ta'(t), \tag{8}$$
$$-s = \varphi(v) + \alpha(\tau) - \tau\alpha'(\tau),$$

where because of (7) could be written

$$a(t) - ta'(t) = \alpha'(\tau), \tag{9}$$
$$\alpha(\tau) - \tau\alpha'(\tau) = a'(t).$$

The corresponding equation of state and the latent heat are determined by

$$p = -f'(v) - t\varphi'(v), \qquad d = -t\varphi'(v), \tag{10}$$
$$\pi = -\varphi'(v) - \tau f'(v), \qquad \lambda = -\tau f'(v).$$

This example applies to any fluid such that the internal energy is the sum of a function of v only and a function of t only, and includes as special case a gas of the Van der Waals type, obtained by putting $f(v) = -a/v$, $\varphi(v) = -R \log (v - b)$.

Similar relations and interpretations for other simple systems, such as electro-chemical cells, black radiation, and capillary films, need only be suggested.

CHARACTERISTIC FUNCTIONS.

The existence of the reciprocity here outlined explains some of the curious analogies that occur in groups of formulas concerned with characteristic functions in the sense of Massieu and Gibbs. In fact, from a group of relations expressed in terms of a characteristic function in any set of variables can be obtained a similar group for the dual set of variables in terms of the dual function. In particular, if the representation

of the properties of a system be given by means of a thermodynamic potential of the energy type most commonly used, there results at once the dual representation by means of a function of the nature of entropy, and conversely. Fuller treatment of this application of the principle is reserved for another paper; the following examples illustrate the computations required.

Let $\psi(v, t)$ be the free energy of a system having the single configuration coördinate v. Then the familiar formula $s = -\partial\psi/\partial t$ yields $u = \partial/\partial\tau(\psi/t) = -t^2\partial/\partial t(\psi/t)$, which reduces to $u = \psi - t\partial\psi/\partial t$. Also, if p be the generalized force corresponding to v, let $\zeta(p, t)$ be the Gibbs function so denoted, whose dual function ζ/t is to be considered expressed in terms of π and τ. This transformation requires the operations

$$\frac{\partial}{\partial\tau} = -t\left(t\frac{\partial}{\partial t} + p\frac{\partial}{\partial p}\right), \qquad \frac{\partial}{\partial\pi} = t\frac{\partial}{\partial p}.$$

Then the relation $s = -\partial\zeta/\partial t$ has for dual $u = \partial/\partial\tau(\zeta/t)$, which when interpreted in terms of p and t is $u = \zeta - t\partial\zeta/\partial t - p\partial\zeta/\partial p$. Similar pairings occur in other formulas, although the duals of known formulas may not be familiar. The extension to any number of variables is immediate.

IRREVERSIBLE PROCESSES.

To adapt the scheme to phenomena involving irreversibility let $\delta'q$ and $\delta'w$ represent the actual heat and work, while δq and δw are the reversible values corresponding to the same changes of intrinsic energy and entropy. Let $\delta_0 s$ be the uncompensated increase of entropy, the part of ds that does not enter with heat communicated. Then $\delta'q - \delta q = \delta'w - \delta w = -t\delta_0 s = \delta_0 u$ say, where then $\delta_0 u$, necessarily negative, is that part of the increment of u which is due to the failure of entry of heat in amount $\delta q - \delta'q$, and marks the departure of the actual from the reversible process. The definitions suitable here are

$$\delta'e = -(du - \delta_0 u)/t = \delta e + \delta_0 u/t, \qquad (11)$$
$$\delta'r = \delta'w/t = \delta r + \delta_0 u/t,$$

and combined with the previous relations give

$$\delta'q = \delta q + \delta_0 u, \quad \delta'w = \delta w + \delta_0 u, \quad \delta_0 u = -t\delta_0 s, \qquad (12)$$
$$\delta'e = \delta e - \delta_0 s, \quad \delta'r = \delta r - \delta_0 s, \quad -\delta_0 s = \tau\delta_0 u,$$

showing that the previous reciprocity is maintained. Conditions of equilibrium and stability, resulting chiefly from the fact that for real processes $-\delta_0 u$ and $\delta_0 s$ must be positive, can therefore be stated in the dual forms. In particular, the two general criteria of stability given by

Gibbs can have their known mutual dependence indicated in terms of the transformation here developed.

It should be noted, however, that two situations theoretically dual may contrast widely in a practical sense, partly because of differences in order of magnitude or limitations of instrumental technique, chiefly because transfers of energy are under better control than changes of entropy. In default of fuller knowledge one is compelled to regard $\delta_0 s$ as entropy actually generated within a system, while ordinarily no such un-traced generation of energy is admitted. Still, in the case of radio-active matter there occur changes of energy tantamount to this from the point of view of the experimental identification of energy values by cyclic processes.

THE UNIVERSITY OF CHICAGO,
December 3, 1919.

ON SOME APPLICATIONS OF HERTZ'S THEORY OF IMPACT.

By C. V. Raman.

Synopsis.

Statement of the Theory.—One of the premises on which the mathematical theory of the collision of elastic solids given by Hertz is based is that the strains produced in the immediate neighborhood of the region of contact are determined by the pressure subsisting at any instant between the bodies, and are practically the same as under statical conditions. This premise is valid even when the impinging bodies do not move as rigid bodies, and the impact results in part of the translational kinetic energy being transformed into energy of elastic wave-motion in the substance of the solids. *Hertz's theory of impact* with suitable modifications may accordingly be applied under a very wide variety of conditions. In the present paper, an attempt is made to discuss the problem of *the transverse impact of a solid sphere on an infinitely extended elastic plate of finite thickness* and to calculate the theoretical *coefficient of restitution* which is a function of the elastic constants and densities of the materials, the diameter of the sphere and the thickness of the plate, and of the velocity of impact. As the result of the impact, annular waves of flexure are set up in the plate, and the sum of the kinetic and potential energies of the wave-motion may be determined in terms of the magnitude and duration of the impulse on certain simplifying assumptions. The calculation results in a *simple formula for the coefficient of restitution.*

Experiments.—A series of *experiments* carried out in the author's laboratory by Mr. A. Venkatasubbaraman has furnished a quantitative confirmation of the formula within the limits of its applicability, that is for plates not thinner than about half the diameter of the spheres. For plates much thinner than this, theory and experiment agree in indicating a zero coefficient of restitution. The formula indicates that the coefficient of restitution should increase and approach unity for greatly diminished velocities of impact, and this is also confirmed in experiment. The paper concludes with indications of some *further applications and extensions* of Hertz's theory of impact.

1. Introduction.

As is well known, Hertz[1] developed a solution of the problem of the collision of curved elastic solids on the following premises: (1) the elastic state of the two bodies near the point of impact during the whole duration of impact, is very nearly the same as the state of equilibrium which would be produced by the total pressure subsisting at any instant between the bodies, supposing it to act for a long time; (2) it is further assumed that the time of impact is large compared with the time taken by elastic waves to traverse the impinging bodies from end to end,

[1] Hertz's Miscellaneous Papers, English edition, page 146, and Love's Theory of Elasticity, page 195 (1906 edition).

which consequently move practically as rigid bodies except in the immediate neighborhood of the region of contact. From these premises, it follows at once that the energy of the colliding bodies remains as translational energy after the impact, a deduction which is closely borne out in experiment, provided the impinging bodies are of appropriate shape, *e.g.*, solid spheres, and the stresses set up do not transcend the limits of perfect elastic recovery.[1] The first of the two premises on which Hertz's theory is based is of very general validity, in as much as it depends for its truth on the consideration that the elastic deformations near the region of contact are determined mainly by the pressure subsisting between the bodies at the instant, and being of a local character and relatively large, are un-influenced by any changes in the elastic deformations that might be developed elsewhere as the result of movements of the bodies. The second premise of Hertz is however purely an assumption[2] which is of comparatively restricted validity and may even fail completely. Indeed it is often the case that the colliding bodies *cannot* even approximately be considered to move as rigid bodies during and after the collision, and a considerable proportion of the energy is transformed into the energy of elastic wave-motion set up in the substance of the solids. Hertz remarked in his paper, though he did not fully develop the idea, that even in such cases, the first hypothesis (which remains valid) taken together with the equations of wave-motion in an elastic solid might enable the course of the impact to be traced. This suggestion of Hertz does not appear to have been generally followed up, though indeed in one case, that of the longitudinal impact of rods with rounded ends, its utility has been established.[3] It is proposed in the present paper to consider the application of the method suggested by Hertz to the problem of the *transverse* impact of a sphere or other solid of limited dimensions upon an infinitely extended elastic plate of finite thickness. It will be shown how the proportion of energy of impact transformed to energy of elastic wave-motion may be approximately calculated, in other words how the coefficient of restitution of the impinging body may be theoretically determined.

2. On the Nature of the Wave-Motion set up by Impact.

The effect of an impulse of short duration applied at a point on the plane face of an infinite mass of elastic solid has been investigated by Lamb,[4] who found that the main shock produced by the impulse travels

[1] C. V. Raman, PHYSICAL REVIEW, Dec., 1918, page 442.

[2] Except in the case of *extremely small* velocities of impact when it is true irrespective of the shapes of the impinging bodies.

[3] J. E. Sears, Trans. Camb. Phil. Soc., Vol. XXI (1908), page 49.

[4] Phil. Trans. Roy. Soc., A, Vol. 203 (1904), pages 1–42.

along the surface of the solid as a solitary wave (with one maximum and one minimum, both in the horizontal and vertical displacements), with its time-scale constant, and its amplitude decreasing in accordance with the usual law of annular divergence, so that its total energy remains undiminished. The velocity of this solitary wave is that of the free Rayleigh waves on the surface of the solid which is somewhat less than that of the equivoluminal waves in an unlimited medium. As the depth to which the Rayleigh waves penetrate is comparable with their wave-length, it may be readily shown that for the case of a sphere impinging on the plane face of an infinite solid, the fraction of the translational energy transformed to energy of elastic wave-motion is extremely small. When, however, the impact takes place on an unlimited elastic plate or bar of *finite* thickness, this is no longer true, and a considerable proportion of the energy (in some cases, the whole of it) may be so transformed. The theory of wave-motion on a cylindrical rod of infinite length has been discussed by Pochhammer in a well-known memoir,[1] and for the case of an infinite elastic plate of finite thickness by Lamb[2] in a recent paper. Lamb finds that the types of wave-motion possible in an elastic plate may be divided into two classes, (*a*) the symmetrical modes, and (*b*) the asymmetrical modes. The former class travel with very high velocities ranging from a maximum equal to the highest value possible in an unlimited medium to a minimum equal to that of the Rayleigh surface-waves. The asymmetrical modes of wave-motion have relatively smaller velocities ranging from a maximum equal to that of the Rayleigh waves to a minimum value which tends to zero for very long flexural waves. *Prima facie*, it is clear that the modes of wave-motion excited by transverse impact would be chiefly of the asymmetrical class, and that, even of the latter, those chiefly concerned in taking up the momentum of the blow would be the relatively slowly-moving waves. Which particular type of wave-motion preponderates would obviously depend on the duration of impact and the manner in which the pressure exerted by the impinging body varies during impact.

An approximate calculation of the energy taken up by the wave-motion excited in the plate may be founded on the simplifying assumption (which may be justified at least as a first approximation) that the disturbance set up by the impact travels outward in the plate with the velocity of flexural waves having a half-period equal to the duration of impact. Let $2f$, and $2b$ be the thickness of the plate, and the diameter of the sphere respectively, ρ_1, ρ_2 their densities, q_1, q_2, their Young's

[1] Crelle, Vol. 81, page 324, see also Love's Elasticity, 1906, page 275.

[2] Proc. Roy. Soc., Vol. 93 (A series), 1917, page 114.

Moduli, σ_1, σ_2 the values of Poisson's ratio, and τ the duration of impact. The velocity V of long flexural waves of wave-length λ in the plate is given by the formula

$$V^2 = \frac{4}{3} \cdot \pi^2 \cdot \frac{f^2}{\lambda^2} \cdot \frac{q_1}{\rho_1(1 - \sigma_1^2)}. \tag{1}$$

On the foregoing assumption, the radius a of the circle on the plate over which the disturbance has spread at the termination of the impact is given by the relation $a = V\tau = \lambda/2$. Accordingly, we have

$$a^2 = \pi\tau f \sqrt{q_1/3\rho_1(1 - \sigma_1^2)}. \tag{2}$$

The next step is to find the kinetic and potential energies of the wave-motion in the plate. The kinetic energy may be determined if we know the transverse velocity of the plate at each point over the circle of radius a covered by the wave. The problem is one of two-dimensional wave-propagation analogous to that treated by Lamb,[1] who has discussed the configuration of the annular solitary wave diverging from a point at which a local pressure is applied, rising from zero to a maximum and falling again to zero. Lamb has given a sketch in the paper cited, showing the form of the wave when it has moved out to a considerable distance from the origin, from which it is seen that the wave consists of two parts: a rising part in which the transverse velocity of a point over which the wave passes increases quickly from zero to a maximum positive velocity and decreases again to zero, and a falling part which consists of an infinitely extended 'tail' in which the transverse velocity after reaching a certain maximum *negative* value (which is numerically less than in the first part) gradually drops down to zero again. The first part of the wave in Lamb's diagram passes over any specified point in about 5/8 of the duration of the original impulse. The form of the wave when near the origin just after the impulse has ceased would, of course, differ in details from that described above, but may be approximately represented as in Fig. 1 in which the ordinates represent transverse velocities, and the abscissæ are the radial distances from the point of impact. The direction of the impact is shown in the figure by an arrow.

3. Calculation of the Coefficient of Restitution.

The kinetic energy contained in the wave is given by

$$E = \int_0^a \tfrac{1}{2} \cdot 2f\rho_1 \cdot 2\pi r v^2 dr, \tag{3}$$

where v is the transverse velocity at any point as given by the graph

[1] Proc. Lond. Math. Soc., Vol. 35, page 141 (1902).

in Fig. 1. We may, without appreciable error, denote the sum of the potential and kinetic energies in the wave by twice the integral in (3), that is by $2E$. The impulse given by the impinging body to the plate is given by

$$I = \int_c^a 2f\rho_1 \cdot 2\pi r v dv, \tag{4}$$

the integration in which is to be carried out having due regard to the

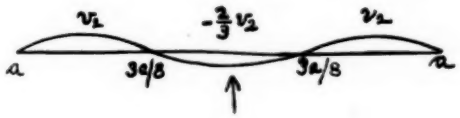

Fig. 1.

sign of v. If M be the mass of the impinging body, v_1 its velocity before impact, and e the coefficient of restitution, we have on the assumption that the energy is fully conserved, the two relations

$$\left.\begin{aligned} \tfrac{1}{2}Mv_1^2(1 - e^2) &= 2E \\ Mv_1(1 + e) &= I \end{aligned}\right\}. \tag{5}$$

On evaluating the integrals in (3) and (4), and substituting the same in (5), we get the value of the coefficient of restitution, e. The necessary integrations are readily carried out by taking the graph of the transverse velocity in Fig. (1) to be made up of arcs of sine-curves. The formula finally obtained is

$$e = \frac{f\rho_1 a^2 - 0.56M}{f\rho_1 a^2 + 0.56M}. \tag{6}$$

The distribution of transverse velocity shown in Fig. (1) is, as explained above, based on Lamb's investigation of two dimensional wave-propagation, and has thus theoretical justification. It also appears to be that

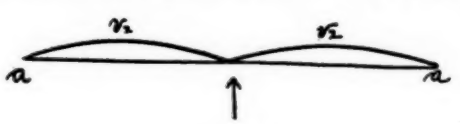

Fig. 2(*a*).

most closely agreeing with facts. Nevertheless, it is of interest to see how far other assumed distributions of velocity would modify the formula for e given in (6). If the distribution of transverse velocity were that shown in Fig. 2(*a*), the formula for e is found to be

$$e = \frac{f\rho_1 a^2 - 0.39M}{f\rho_1 a^2 + 0.39M}. \tag{7}$$

For the distribution shown in Fig. 2(*b*), the formula is

$$e = \frac{f\rho_1 a^2 - 0.44M}{f\rho_1 a^2 + 0.44M}. \tag{8}$$

To make use of the formula given in (6), we have to substitute in it the value of a as given by (2), and to enable this to be done, we have to ascertain the duration of the impulse τ. It is obvious that to a first approximation, this may be taken to be the same as that given by Hertz's theory of impact, the mass of the impinging body being M, and

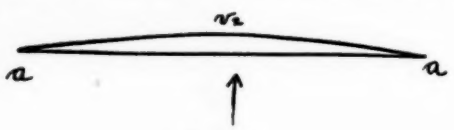

Fig. 2(*b*).

the mass of the plate being taken to be infinitely great. If there is any deviation from this in actual practice, such deviation should be sensible only when the velocity of impact is very large or when the thickness of the plate is much smaller than the diameter of the impinging sphere. The duration of impact on Hertz's theory is given by the equation

$$\tau = 2.94\alpha/v_1, \tag{9}$$

where

$$\alpha = \left[\frac{15}{16}v_1^2\left(\frac{1-\sigma_1^2}{q_1} + \frac{1-\sigma_2^2}{q_2}\right)M\right]^{2/5} b^{-1/5} \tag{10}$$

and b is the radius of the impinging sphere. These values have to be substituted in equations (2) and (6) above.

4. Comparison with Experiment, and Some Concluding Remarks.

The correctness of the formula for the coefficient of restitution developed in (6) above has been tested in the author's laboratory in a very careful series of experiments carried out by Mr. A. Venkatasubbaraman to whom his best thanks are due. The impacts observed were those of polished hard steel spheres impinging on horizontally held glass plates of sufficient size to permit of the application of the theory, these materials being chosen as for moderate velocities of impact they very nearly satisfy the condition that the system is of the conservative type. As remarked above, the calculations assume that the duration of impact is the same as that given by Hertz's formulæ which would be practically correct, provided the velocity of impact is not very great and that the thickness of the plate is not very small. Mr. Venkatasubbaraman's experiments give results for the coefficient of restitution under these conditions closely agreeing with those found from (6).

TABLE I.

Coefficient of Restitution, e.

Velocity of impact = 234 cms. per second.

Thickness of Plate in Centimeters.	Diameter of Spheres in Centimeters.													
	0.791.		0.714.		0.637.		0.555.		0.396.		0.314.		0.237.	
	Calc. *e*.	Obsd. *e*.	Calc. *e*.	Obsd. *e*.	Calc. *e*.	Obsd. *e*.	Calc. *e*.	Obsd. *e*.	Calc. *e*.	Obsd. *e*.	Calc. *e*.	Obsd. *e*.	Calc. *e*.	Obsd. *e*.
2.53.....	0.98	0.95	0.99	0.96	0.99	0.97	0.99	0.97	0.99	0.98	0.99	0.98	0.99	0.98
1.93.....	0.97	0.95	0.98	0.96	0.98	0.96	0.99	0.97	0.99	0.97	0.99	0.98	0.99	0.98
1.29.....	0.93	0.91	0.94	0.93	0.96	0.93	0.97	0.94	0.98	0.95	0.99	0.97	0.99	0.98
1.01.....	0.89	0.87	0.91	0.89	0.93	0.91	0.95	0.93	0.97	0.94	0.98	0.96	0.99	0.97
0.71.....	0.79	0.77	0.83	0.80	0.86	0.83	0.89	0.86	0.94	0.90	0.96	0.95	0.98	0.96
0.49.....	0.61	0.60	0.67	0.66	0.73	0.71	0.79	0.77	0.89	0.86	0.93	0.91	0.96	0.94
0.35.....	0.36	0.38	0.44	0.45	0.53	0.53	0.62	0.61	0.79	0.77	0.86	0.84	0.92	0.89
0.225.....	0	0	0.03	0.13	0.15	0.23	0.28	0.34	0.55	0.55	0.69	0.68	0.81	0.79
0.150.....	0	0	0	0	0	0	0	0.12	0.21	0.28	0.42	0.45	0.62	0.63
0.105.....	0	0	0	0	0	0	0	0	0	0.08	0.09	0.24	0.36	0.43

The following facts emerge on an examination of the figures shown in the table of results. For the thicker plates, the experimental values for e are smaller by two or three per cent. than the theoretical values. This is evidently due to various minor causes of dissipation of energy not being taken into account in the theoretical treatment. For moderate thicknesses of plate, the calculated and observed coefficients of restitution agree well. Theory and experiment also agree in the case of very thin plates in giving a zero coefficient of restitution. In other words, in such cases, the sphere on impact with the plate remains in contact with it. But in certain intermediate cases, where the thickness of the plate is less than about half the diameter of the impinging sphere, but not so small as to give a zero coefficient of restitution, the observed values of e are somewhat larger than the calculated values. These cases are shown enclosed in heavy lines in the columns of the table. It has already been remarked that in such cases, the assumption made that the duration of the impulse is given by Hertz's formula for impact with an infinite mass of solid would cease to be valid, and the discrepancy between the observed and calculated figures may possibly be due to this. An alternative explanation would be that in such cases, the configuration of the annular wave set up in the plate may slightly differ from that shown graphically in Fig. 1, and may approach more closely to that shown in Fig. 2(*a*). It is obvious then that the formula (6) would have to be modified for such cases by slightly decreasing the numerical constant 0.56 appearing in it to 0.50 or 0.45 for the thinnest plates. As a matter of fact, if this is done, the discrepancy between the observed and calculated values of e in these cases disappears.

Another noteworthy result indicated by formulas (2) and (6) and which is confirmed by some observations made by the writer is that the coefficient of restitution for impact on an elastic plate should depend on the velocity of impact, and should approach unity for very small values of this velocity. This is a consequence of the fact that the duration of impact as given by Hertz's formula varies inversely as the one fifth power of the velocity, and the radial distance a covered by the annular wave diverging from the origin should therefore increase with decreasing velocity of impact. Fuller quantitative data showing the relation between the coefficient of restitution and the velocity of impact will be obtained and presented in due course in a further communication.

It may be remarked that much higher velocities of impact or larger spheres than those indicated in the table cannot be used, for the reason that the impact in such cases ceases to be of the conservative type, and results in internal fractures of a local character and of peculiar geometrical form in the glass plates. The character of these fractures bears an interesting relation to the distribution of stress in impact as given by Hertz's mathematical theory, and will be more fully discussed in a later paper.

The method used in the present paper can of course be applied also to the problem of transverse impact on an elastic bar, which is of a somewhat simpler character owing to the wave-propagation being in a single dimension instead of in two dimensions as in an elastic plate. The nature of the wave-motion set up by impact in this case is also capable of somewhat stricter analytical treatment if, following Boussinesq, we use Fourier's well known integral for the transverse vibration of an infinitely long bar to find the motion resulting from the initial impulse. In both cases also, it would be of interest to attempt a theoretical treatment (with experimental verification) of the manner in which the duration of impact varies with the thickness of the bar or plate, and also a direct experimental determination of the form of the wave at the instant at which the impact ceases.

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WAVE-LENGTH MEASUREMENTS IN THE *M* SERIES OF SOME HIGH-FREQUENCY SPECTRA.

BY J. C. KARCHER.

SYNOPSIS.

Wave-length Measurements in the M Series of Some High Frequency Spectra: a New Vacuum X-ray Spectrograph.—The paper contains a description of an X-ray spectograph designed to be used in the same chamber containing the source of X-rays, also a description of the *vacuum chamber*, the *method of sealing*, and *details of manipulation* of the apparatus.

A method is described of determining the position of the photographic plate upon which the lines are photographed.

Experimental data. Some measurements are made of lines in *the M series of bismuth, lead, thallium, mercury, gold, and platinum.*

AS only a small amount of work has been done in the measurement[1] of wave-lengths, in the *M* series of X-ray spectra, the writer was led to attempt some of these measurements by the photographic method. The *M* series lines are almost entirely absorbed by the walls of an ordinary X-ray tube, and even, suffer considerable absorption in passing through 20 or more centimeters of air, hence it is necessary to arrange the X-ray source and spectrograph so that the amount of material in the path of the X-rays is reduced to a minimum. Accordingly a spectrograph was devised that could be used in the same chamber with the X-ray generator. In this way the amount of absorption of the rays, was sufficiently reduced to make photography of the lines feasible.

A diagram of the spectrograph, together with the chamber and parts of the X-ray generator, is shown in Fig. 1. The spectrograph is constructed of brass with the exception of the lead strip *hh* and the iron armature *a*.

The dimensions are apparent when it is noted that the distances from the axis of rotation of the crystal to the photographic plates *gg'*, and the lead strip *h*, are 10 cm. and the angles made by the normals to these plates with the plane through *s* and *k*, are 30°, 90° and 150°. The plates *c* and *d* are 2 cm. apart, and are held in position by means of screws and posts. Slots in *c* and *d* receive the photographic plates. A brass plate slid behind each photographic plate holds it in position by means of springs. A piece of "carbon" paper 0.006 cm. thick is placed in front

[1] M. Sieghahn, Jahr. d. Radioakt, 13, 1916.

of the plate to absorb the ordinary light. This was found to be quite opaque to ordinary light and showed no appreciable absorption of the X-rays in the region investigated. This is the only material which the rays had to penetrate before reaching the plate.

The slits *s* and *s'* are adjustable. The opening *s* was about 0.2 mm. in width, and *s'* about 1 mm. wide, when photographs were taken.

The crystal is supported on the crystal mounting shown in Fig. 2; it is rotated in the chamber *l*. The position of the crystal is indicated approximately by a pointer moving along the scale *p*. Rotation is produced by means of a magnet (not shown) placed beneath the glass plate. The rotating magnet actuates the armature *a* which rotates the crystal through a gear train with a speed reduction of 5,000 to 1.

The cathode and water-cooled anode are housed in a glass shield *t* which has a 2 cm. opening "*o*" between the anode and slit *s*. The face of the anode consists of a piece of electrolytic copper, into which a specimen of the element being investigated, is imbedded. Each specimen is mounted on a separate piece of copper. The mercury specimen had to be frequently renewed because of evaporation.

The whole is mounted on a glass plate 50 cm. × 50 cm. × 3.1 cm. thick, then covered by a bell jar 40 cm. in diameter. The joint between the jar and plate is sealed by means of vacuum wax and mercury. A strip of "fiberboard" *n* is placed on edge about the edge of the jar and about 1 cm. distant thus forming a reservoir for the mercury.

Before placing the jar on the plate, a thread of vacuum wax is run around the edge of the jar, the jar is then set in place and the annular reservoir is filled to a depth of about 2 cm. with mercury. When it is desired to open the chamber, after the vacuum has been relieved, a two-way connection on the suction side of the supporting pump, and suitable traps, permit the mercury to be quickly removed from the reservoir by suction, after which the jar can be lifted from place.

Evacuation was accomplished by means of a Gaede Molecular pump supported by a single-stage oil pump. The pressure in the chamber could be reduced from atmospheric to that required for the operation of the X-ray generator, in about 12 minutes. The vacuum was regulated by varying the speeds of the pumps.

The source of power for the X-ray generator consisted of a transformer operating on 60-cycle alternating current and a Coolidge tube used as a rectifier. The potential drop between the electrodes of the X-ray generator was kept sufficiently low so that the *L* series of the metal under investigation did not appear. This was necessary to prevent confusion of the weak lines of the *M* series with the strong lines of the *L* series in the third and fourth orders.

Exposures were made by rotating the crystal at the rate of 1° to 1.°25 per hour, while a current of 6 milliamperes was flowing through the X-ray generator. A selenite crystal was used for the measurements here noted.

Since all the lines photographed were on plates in the position (*gg*, Fig. 1), the angular position of the plates was determined by the following simple method: thus

$$N\lambda = 2d \sin\frac{\theta}{2}.^{1} \tag{1}$$

If a line appears on the plate in both the first and second order, we have

$$2 \sin\frac{\theta_1}{2} = \sin\frac{\theta_2}{2}. \tag{2}$$

Where $\theta_1/2$ and $\theta_2/2$ are the glancing angles for the first and second order reflections respectively.

Also (Fig. 3)

$$\theta_1 = \alpha + \tan^{-1}\left(\frac{x_1 - a\tan\alpha}{a}\right) \tag{3}$$

and similarly:

$$\theta_2 = \alpha + \tan^{-1}\left(\frac{x_2 - a\tan\alpha}{a}\right), \tag{4}$$

which reduce to

$$\text{Cot. } \theta_1 = \frac{a}{x_1 \cos^2\alpha} - \tan\alpha, \tag{5}$$

$$\text{Cot. } \theta_2 = \frac{a}{x_2 \cos^2\alpha} - \tan\alpha. \tag{6}$$

Where x_1 and x_2 are the respective distances of the first and second order lines from the o line, a is the perpendicular distance of the plate from the axis of rotation of the crystal and α is the angle between the perpendicular line along which a is measured and the plane passing through the slit and axis of the crystal.

From (2), (5) and (6), we have

$$2 \sin\left[\tfrac{1}{2}\,\text{Cot}^{-1}\left(\frac{a}{x_1\cos^2\alpha} - \tan\alpha\right)\right] = \sin\left[\tfrac{1}{2}\,\text{Cot}^{-1}\left(\frac{a}{x_2\cos^2\alpha} - \tan\alpha\right)\right], \tag{7}$$

which expresses α in terms of x_1, x_2 and a. Since α is approximately known, a graphic solution of (7) will easily determine α to the nearest 0.1 min. If an average of several determinations is taken, the probable error in a wave-length measurement, due to error in α will be less than

[1] W. H. and W. L. Bragg, Proc. Roy. Soc., A, 88, 1913, p. 428.

the probable error due to error in x for that measurement. The L Series lines of tin shown in Fig. 4 were used in the determination of α.

The M series spectrum was photographed from one to four times for each of the elements, bismuth, lead, thallium, mercury, gold and platinum. The results of the measurements are given in the following table. As many as 12 lines were found on a plate in the region of the M Series but only those which were sufficiently identified are noted. Measurements on the plates were made with the aid of a traveling microscope.

Constants entering into the computations are:

$$a = 10.030 \text{ cm.}, \quad \alpha = 29^\circ\ 27.'6,$$

$$\log 2d = 1.18300 \text{ for selenite.}$$

λ *in Ångstrom Units.*

Element.	At No.	α_1.	β_1.	β_3.	γ_1.	γ_2.	γ_3.
Bi.....	83	5.124	4.915	[1]4.604	4.534	[1]3.932	[1]3.840
Pb.....	82	5.290	5.078		4.675	[1]4.073	
Tl.....	81	5.468	5.254				
Hg.....	80	5.649	5.439				
Au.....	79	5.848	5.632	[1]5.446	5.154	[1]4.530	[1]4.439
Pt.....	78	6.049	5.831	[1]5.649	5.329	[1]4.733	[1]4.623

[1] New lines.

The work is being continued.

I am indebted to the University of Pennsylvania for the use of apparatus.

RANDAL MORGAN LABORATORY,
UNIVERSITY OF PENNSYLVANIA,
PHILADELPHIA, PA.

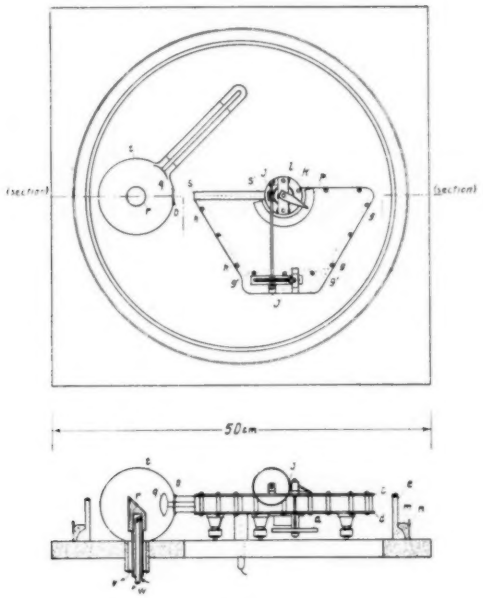

Fig. 1.

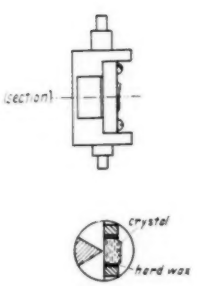

Fig. 2.

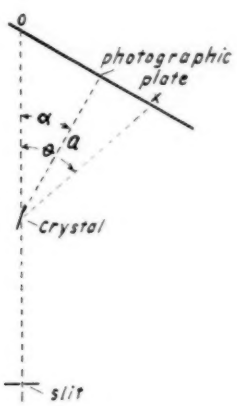

Fig. 3.

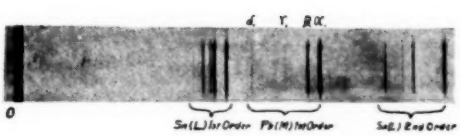

Fig. 4.

Spectrum of Lead-tin Alloy.

J. C. KARCHER.

"THALOFIDE CELL"—A NEW PHOTO-ELECTRIC SUBSTANCE.

BY T. W. CASE.

SYNOPSIS.

A new photo-electric cell is described which is composed of thallium, oxygen and sulphur. This cell shows maximum sensitiveness to light of wave-length about 10000 Angstrom units. Some of the best cells lower their dark resistance by fifty per cent. in .06 foot candles when the light source is a tungsten filament. The dark resistance of different cells may be from 5 megohms to 500 megohms. The induction and deduction periods of the cells are very small. The chief characteristics of the cell are mentioned.

IN the April issue of the PHYSICAL REVIEW, 1917, the author published a list of substances which were examined for change of resistance under the influence of light. This work was continued, although no substance at that time had been found which compared in sensitivity with selenium. Most of the new combinations found, however, indicated that sulphides would be the most promising field for future work.

On October 2, 1917, the author found that thallium sulphide (fused) showed a slight change of resistance under the influence of light.

After two years of research upon this compound, very sensitive cells have been made to which the name "Thalofide Cell" has been given, indicating that the present sensitive substance is composed of thallium, oxygen and sulphur. The thalofide material after careful preparation is first fused on a $\frac{3}{4}''$ quartz disc and then it is placed in an evacuated tube. The effect of the vacuum is to increase the sensitivity of the element from three to five times and also preserve its life by preventing oxidation. Several hundred of these cells are now over a year old and have not lost any of their initial sensitiveness.

As no quartz spectrometer was available at the beginning of this work, the spectral sensitivity was roughly determined by means of combinations of Wratten filters; and the maximum sensitiveness was found to be somewhere between 8,000 Å.U. and 15,000 Å.U. Since that time, Dr. W. W. Coblentz of the Bureau of Standards has accurately determined the maximum point at 10,000 Å.U. The spectral sensitivity curve of this cell in a red glass bulb is reproduced in Fig. 1.

By means of the same Wratten filters, it was found that the blue and violet rays had a deleterious effect on the element in that after

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rather strong exposure, the resistance was somewhat permanently lowered by this upper part of the spectrum, and the recovery period after exposure was much slower. This was prevented by the use of

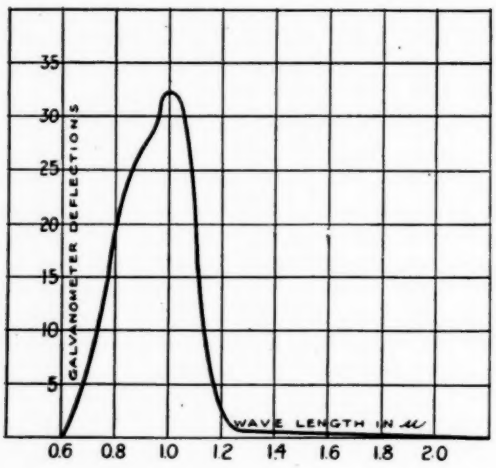

Fig. 1.

flashed copper ruby glass for the material of the vacuum bulb. The cells can then be exposed to much stronger radiation without harm, and it was also found that the recovery period was extremely fast, which is in marked contrast to selenium. The average sensitivity of these cells

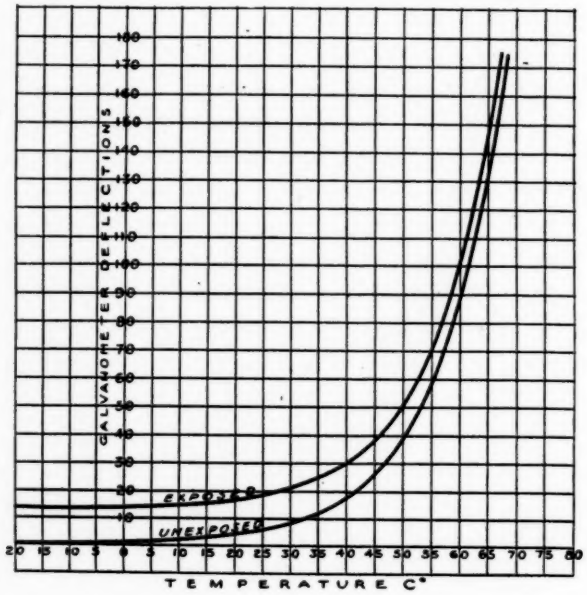

Fig. 2.

is such that their dark resistance is lowered by fifty per cent. in 0.25 foot candles when the source is a tungsten filament. Some of the best ones however, will lower their dark resistance by fifty per cent. in 0.06 foot candles.

These cells have been designed so that a large per cent. action is obtained between dark resistance and resistance on exposure, consequently their dark resistance is high as compared to the ordinary selenium cell. The resistance in the dark of different cells may range from 5 megohms to 500 megohms.

The dark resistance varies inversely with the temperature. A char-

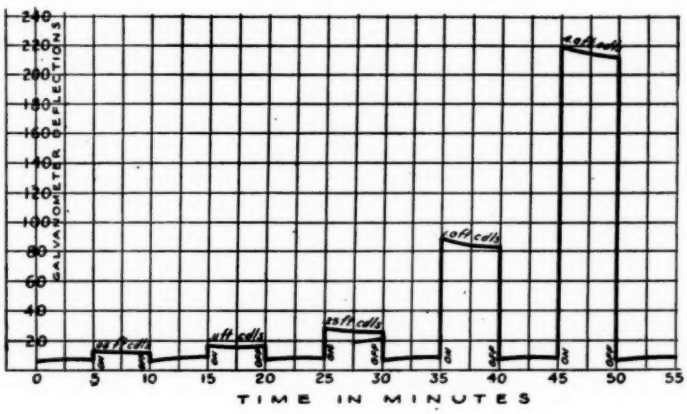

Fig. 3.

acteristic temperature curve together with light action of a cell on a fixed potential is shown in Fig. 2.

As a general rule, not over 50 volts should be used on these cells. On applying the potential, the resistance will be gradually lowered but an equibrium is soon reached. On exposure to light, the resistance is immediately lowered and upon cutting off the light, the original resistance is almost instantly reached if a very low intensity has been used. If a very sensitive galvanometer is at hand, a lower applied voltage is more satisfactory. Fig. 3 shows a characteristic curve of one of these cells on a potential of 10 volts when exposed to various light intensities. These curves vary with the individual cell used, the light intensity and the applied voltage. Each cell should be studied under the conditions for which it is to be used.

For measuring much more intense radiation than that of stellar magnitude, the author suggests the reduction of the exposure to a fraction of a second in order to do away with any time change factor. A camera shutter operated at 0.01 or 0.02 of a second will give a good reading

on a ballistic galvanometer and by comparing the throws of the source to be measured with the throws of a known calibrated source, very accurate results may be obtained. It is suggested that this method be tried for very high temperature measurements.

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NOTE ON ARTICLE BY H. M. DADOURIAN ON "SOFT X-RAYS."

BY ELIZABETH R. LAIRD.

SYNOPSIS.

Soft X-Rays, (1) Criticism of Use of Electrostatic Trap to Avoid Methodical Errors of Observation.—Unless the field is fully symmetrical one kind of ions or of rays will get through, and light is not thus trapped.

(2) *Criticism of Measurements of Photoelectron Velocities.*—The beforementioned sources of error are present, and the use of a secondary field renders uncertain the value of the supposed retarding field.

(3) *Criticism of the Values Given for Variation of Intensity of Radiation with Voltage, and for Absorption in Celloidin Films.*—Errors as in (1) persist. Experiments are described exhibiting similar variations the incorrectness of which is shown by a comparison of the photoelectric and ionization currents, and by observations with the apparatus further denuded of gas. A *discussion* is given of the use of the term soft X-rays, and the *suggestion* is made that the intensity of radiation in this region from a solid produced by the impact of cathode rays of less than 200 volts velocity is small.

IN the recent article by Dr. Dadourian[1] on "Soft X-Rays" he uses the term "to denote roughly that region of general radiation which falls between the shortest known ultra-violet rays and the longest waves of characteristic X-radiation studied to date" and claims to have produced such radiation by cathode rays of from 20 volt velocity up. I should like to question the nature of his proof.

The apparatus used, in so far as it consisted of three parts, a discharge tube, an electrostatic trap, and a measuring chamber, is essentially the same as that described earlier by me,[2] the differences being that my early discharge tube was of glass, and that the field applied in the middle chamber was a radial one. Dadourian's first proof of the correctness of his results is that after a certain value of the field between his plates P and P', Fig. 1 (L.C.) is reached, the effect measured becomes constant as shown by the horizontal parts of the curves in his Fig. 3. In spite of this there might yet be ions, positive rays, and light entering his measuring chamber. No statement is made as to the vacuum obtained with the apparatus, but with the curve at 80 volts, for example, the putting on of a 10-volt field in his trap reduces the effect measured by

[1] H. M. Dadourian, PHYS. REV. (2), 14, 234, 1919.

[2] E. R. Laird, Ann. der Phys., 46, 605, 1915.

95 per cent., and by almost as much when a celloidin screen is used. This indicates a plentiful supply of ions in the apparatus, and as the field is increased the original ions may produce fresh ions in the field, some of which will pass on through the next opening. The sign of these would depend on the sign of the potential applied to the plate P, as the field is evidently not symmetrical, P' being connected to the case and the partition in which occurs the opening to the measuring chamber. Nothing is said as to whether the field was reversed. If P were $+ ve$, then fast-moving positive ions would naturally go from the anticathode to the opening S into the trap and some almost certainly on into the final chamber. I have already described a case where such positives after passing a thin collodion window not quite airtight will charge an electrode already positive. The more serious point in this case is probably that with a brass discharge tube, and a platinum anticathode, and vacuum as indicated, there would be present light of wave-length greater than 1,000 Å. capable of producing a photoelectric effect, which of course would not be diminished by the application of an electric field.

Dadourian's second argument rests on his attempts to measure the velocity of his photoelectrons. In Fig. 4 he shows a curve obtained by the retarded field method at 390 volts on the discharge tube. At this potential the presence of an X-radiation is not doubted, though it may be accompanied by light and other effects. I have previously shown a curve[1] (Fig. 6, Curve II.) at 550 volts not intersecting the axis until 20 volts retarding potential is reached, as compared with his 5 volts. My apparatus was evidently more favorable as the airtight window kept out ions or positive rays. I obtained also the negative saturation part of the curve, and an increase in the retarding potential by use of a secondary gauze. The slow rise to saturation of his curves would be explained by the presence of some fast moving positive ions which as previously suggested are probably present. In his Fig. 5, for 60 volts, where the attempt is made to demonstrate the presence of photoelectrons of velocities greater than 10 volts, the so-called retarding field was made up of two gauzes fairly close together, of 1 mm. mesh, to the inner one of which the negative potentials were applied, while the outer was at + 200 volts. It would be difficult to say what the field between plate and first gauze is under such circumstances.[2] Van der Bijl has shown that an electron current may pass through a gauze to a positive plate even though the gauze be at a negative potential, since the potential of the plate causes a stray field to act through the openings of the gauze.

[1] L. c., p. 616.

[2] Van der Bijl, PHYSICAL REVIEW, 12, 171, 1918.

The magnitude of the factor involved depends on the mesh and the nearness of the plate. Since in Dadourian's experiments + 5 volts sufficed to hold back the great majority of photoelectrons, the use of + 200 on the second gauze makes it highly probably that the effect of this was to counteract the assumed retarding field. The difference between the effects observed with and without blackening the walls of the chamber holds equally for light.

The other data shown, Fig. 7, are for 200 volts. To Curve I. the same criticism regarding the use of + 200 volts on the outer gauze applies. For Curve II. there is nothing to show that this effect would not be produced by light with or without the presence of some ions. In my former article[1] I referred to having made experiments at 200 volts, where without gauzes effects up to 4 volts retarding field were obtained but later I found that 60 per cent. of the radiation was transmitted by fluorite, so that it was concluded that the effect was due to ultra-violet light.

In Part III., Dadourian gives curves showing the relation between the potential on the discharge tube and the effect measured, and indicating the amount transmitted by thin celloidin. Although by his results the transmission of celloidin does not vary greatly between 200 and 800 volts, the form of these curves differs so markedly from those obtained by me[2] with an airtight celluloid window that some explanation should be expected. In Fig. 1, a curve is shown which was obtained with a brass anticathode and with an aluminum plate in vacuum in the measuring chamber, pressure .001 while the discharge was going in the discharge tube. It shows a peak at above 400 volts, and no increase in intensity at 600, also a peak at below 200 volts. That the irregularities were due to the effect of light of wave-length greater than 1,200 Å. was shown by the facts that (*a*) previously a number of tests had shown that a large part of whatever effect was obtained below 200 volts would go through fluorite, (*b*) that if a small amount of air was admitted to the measuring chamber no ionization was found below 200 volts and the photoelectric effect was diminished, (*c*) that above 400 volts where the other irregularities occurred, they practically vanished when air was admitted to the measuring chamber, as the ionization current totally overshadowed any small remaining effect. Curve 2 shows an ionization curve obtained on the same day as the curve I. of Fig. 1. But if these peaks indicated characteristic radiation occurring at this voltage, this radiation should produce ionization and the peaks should be magnified

[1] L. c., p. 617.
[2] L. c.

instead of suppressed. Further with greater effort to rid the apparatus of gas, they are suppressed even in the vacuum measurements.

The very slow rise in Dadourian's curves above 400 volts of itself suggests that the effect measured is largely due to light or to positive rays which become a fairly constant proportion of the original current. It may be noted that the curves of Fig. 3 show apparently a greater proportion of the effect measured transmitted by celloidin at 80 volts

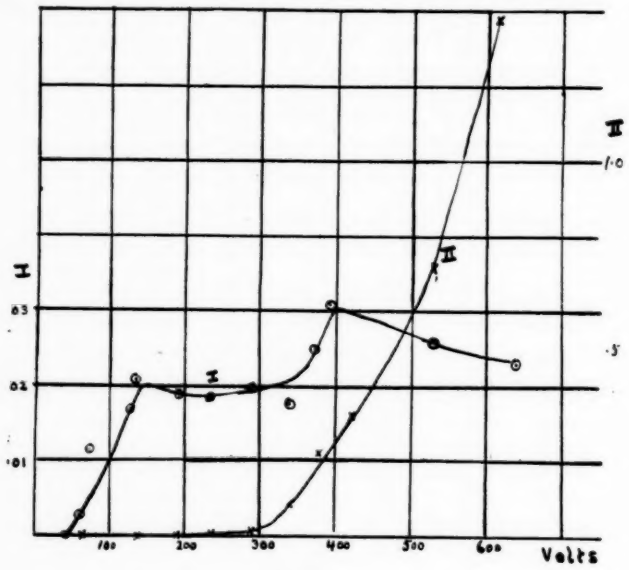

Fig. 1.

than at 380. Contrary to what is given in Fig. 8. Quite evidently as the voltage diminishes towards zero, below some value still to be determined, the absorption in these films would diminish, and not increase.

It is a well-known fact that the impact of electrons of 5 volts velocity and less on the molecules of certain vapors produces a radiation, and that velocities of 10 volts are sufficient to produce ionization in air which in turn would be accompanied by radiation, so that it is not questioned that electrons of less than 200 volts velocity produce radiation by impact, but there appears as yet no proof that such radiation lies in the region designated as belonging to soft X-rays. The properties of " entladungs-strablen " differ somewhat from those of the radiation produced by cathode rays above 300 volts, in a way to suggest longer wave-lengths and more marked absorption changes than found by Dadourian. But while it is not to be doubted that the impact of electrons on a solid will produce under the right conditions a radiation of the desired wave-length, it may be in such small amount as to make the distinguishing between it and other effects very difficult.

SOFT X-RAYS PRODUCED BY CATHODE RAYS OF FROM 200 TO 600 VOLTS VELOCITIES.

By Elizabeth R. Laird and Vola P. Barton.

Synopsis.

Soft X-Rays, Examination of Method of Investigation Using No Window.—Previous results by the one author are briefly referred to, and a new experiment is described showing the difficulties in measuring a photoelectric effect due to soft X-rays produced in the same chamber as that which contains the photoelectric electrode.

Soft X-Rays Produced by Cathode Rays of between 200 and 600 Volts Velocity, Effect of Occluded Gases in the Anticathode on the Intensity.—The *method* used for estimating the radiation produced was to allow the soft X-rays to ionize the gas in a chamber separated from the discharge tube by a thin celluloid window. (*a*) A platinum anticathode was partially denuded of gas by glowing, and (*b*) the remnants of gas in the tube were displaced by hydrogen, the intensity of radiation was not notably altered. The *conclusion* is drawn that the radiation observed comes from the solid anticathode.

Variation of Intensity of Radiation with Voltage for Anticathodes of Brass, Platinum, Graphite, Aluminum, Lithium and for Gas Remnants, Air, Hydrogen, and Carbon Bisulphide.—After showing that the radiation is proportional to the cathode ray current, and that relative measurements could be fairly well repeated, even with different *pressures of gas in the ionization chamber*, the authors give details of the relative intensities and of the actual ionization current at 600 volts. The intensity at 300 volts was less than 2 per cent. that at 600 volts.

Characteristic and General X-Radiation.—From a comparison of (*a*) the radiation from different anticathodes, and (*b*) the intensity as measured by ionization and by the photoelectric effect, it is inferred that the radiation is largely general. There is evidence of a characteristic (K) radiation from carbon, and one observation suggesting a characteristic (L) radiation from aluminum. At the end of the paper are found *suggestions* of possible reasons for the similarity of behaviour of different anticathodes.

Soft X-Rays from 200 to 300 Volts.—It appears that the radiation in this region ionizes the gases used relatively less powerfully in proportion to its photoelectric effect.

X-Radiation below 190 Volts.—Tests are described showing that the effects thus far obtained by the authors were due to gas in the tube and not to a radiation from the solid anticathode.

THE following experiments have been done at odd times in the last three years in continuation of work previously published by one of us[1] on the X-rays produced by slow moving cathode rays. In the previous work it appeared among other results that X-rays were not obtained by the impact of cathode rays of less than 200-volt velocity

[1] Laird, Annalen der Physik (4), 46, p. 605, 1915.

on solid anticathodes, or else that such rays would not pass through a window of celluloid film to produce a measurable photoelectric or ionization effect, that above this voltage such rays were produced, the intensity of which seemed fairly independent of the anticathode used. The attempt to measure the wave-length by the velocity of the photoelectric electrons produced by the rays was not found very satisfactory, the method showing velocities up to not more than 100 volts when the cathode rays were of 500-volt velocity. In seeking to throw light on the questions raised by these results it was proposed (*a*) to make a further attempt to get on without a window, (*b*) to try other methods of ridding the anticathode of gas, supposing this might account for the similar behavior of the different anticathodes, (*c*) to examine more carefully the intensity of the radiation at different voltages, as measured by the ionization currents for comparison with the photoelectric effect and (*d*) to seek for characteristic radiations in the interval 200–600 volts.

The various researches on the emission of electrons from heated tungsten filaments led to the hope that by use of such, observations could be made at high vacuum on soft X-rays without use of a window. For this purpose after some preliminary experimenting on the use of such

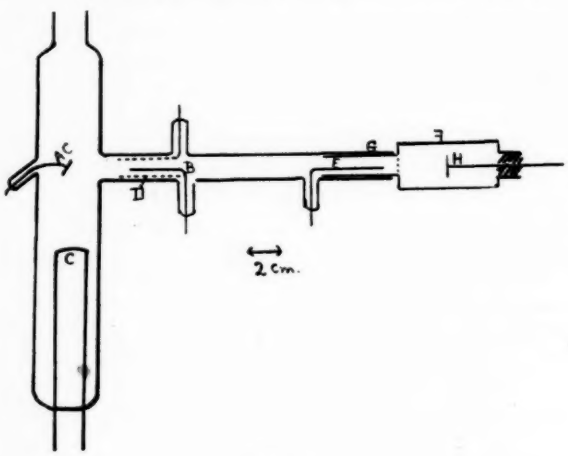

Fig. 1.

filaments, obtaining currents of .06 ampere with no luminosity that could be seen by a casual observer but which could be detected at the instant of putting current on and off, the apparatus shown in Fig. 1 was used. In this all seals were glass seals except at *G* where the metal chamber was sealed on with de Khotinsky cement. *C* was a tungsten wire, *AC*, a small brass anticathode, *D* a gauze, *B*, *E*, wires, *F* a metal cylinder, *H* an electrode connected to an electrometer. The pump used

was a Gaede mercury pump. The tube was heated in an electric oven to 350° for some hours, while pumping, CO_2 and acetone were used on a charcoal tube to freeze out mercury vapor. The pressure while the electron current was going was in the neighborhood of .0001 mm.; observations were then made on electrometer deflections in a given time with a potential difference of 135 volts between cathode and anticathode. It was found that establishing a field between B and D changed the deflections obtained enormously, and that even when such a field was used so that changing the potential of E from 0 to 135 made very slight further change, yet if F were changed from + 5 volts to − 1.3 the sign of the electrometer deflection was changed. This might be due to photoelectric electrons from the edge of the opening into F, but the experience made it improbable that one could establish the existence of an X-radiation at low voltages by this method. This was in agreement with the observations made in the earlier experiments. In this connection it is interesting to note that Richardson and Bazzoni[1] using the method of magnetic deflection for measurement of the velocities of ejected electrons were not able to show the presence of a radiation of wave-length greater than 60 Å. (and did not try for smaller wave-lengths) from a solid anticathode in a high vacuum. Their method however would not be as sensitive as the more simple ones.

Having concluded from the experiments just described that more progress could be made with use of an airtight window, a return was made to the previous arrangement of apparatus and a Wehnelt cathode. The apparatus, which was originally set up by Miss Barton, is shown in Fig. 2. Slight changes were made in it from time to time. Both discharge tube B and measuring chamber A were of brass. The anticathode and plate electrode P were carried by side tubes. P was connected to an electrometer, with amber insulation throughout. The plate P was later changed for a long wire as indicated.

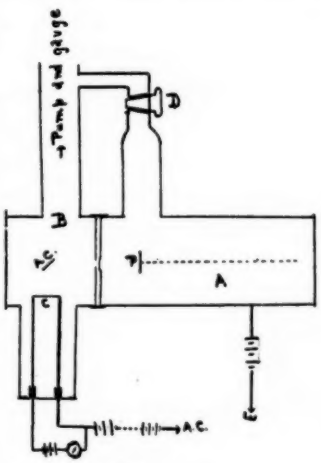

Fig. 2.

The cathode ray current up to potentials of 450 volts was supplied by a combination of dry cells and small storage cells, above this by the addition in the circuit of the line from a small dynamo. P.D. measurements were made at first with a high resistance voltmeter, which as

[1] Richardson and Bazzoni, Phil. Mag., 34, 285, 1917.

used would take about 2 milliamperes current. Later an electroscope clamped in place, the leaf of which was projected by a clamped lens on a scale formed a convenient and inexpensive electrostatic voltmeter. The scale was calibrated by groups of cells, in turn calibrated by a potentiometer. With this the potential on the tube could be read while taking readings with the electrometer. The current through the tube was read on a shunted Leeds and Northrup pointer galvanometer. The heating current on a Weston ammeter graduated to .1 ampere, graduations being about 3 mm. apart, it was furnished by storage cells. The electrometer needle was charged by a small set of cadmiun cells to 80 volts, its sensitiveness with the connections was 3.3×10^{-15} ampere per sc. div. per minute, and was very constant, one third of this was measurable in the best conditions.

Two methods were employed to test whether the X-radiation produced when slow cathode rays fall on the anticathode comes from gas occluded in the anticathode. The first was to introduce a thin platinum foil, which could be electrically heated, as anticathode. A set of readings were taken with air at a pressure of 4 mm. in chamber A, pumping on

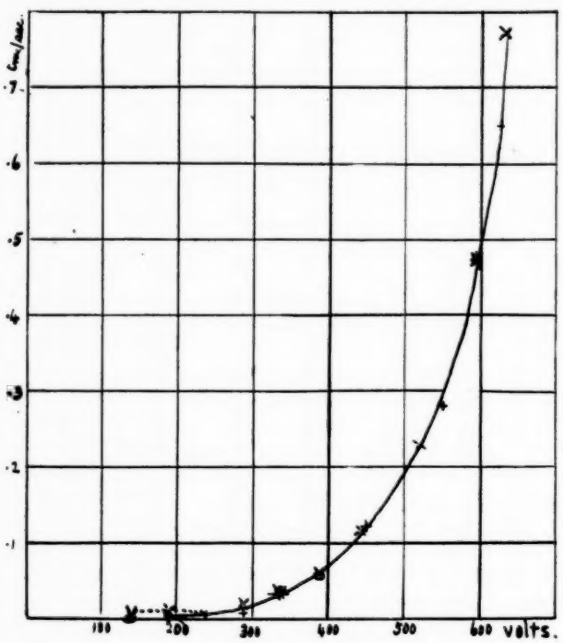

Fig. 3.

the other side to .002 while the discharge was going. The walls of the chamber were + 6 volts. The results are shown in the curve of Fig. 3 by points marked ×. The anticathode was then glowed for an hour

and a half, the center being at white heat the last ten minutes, pumping all the time, during the last hour CO_2 and acetone were used on a charcoal tube. At the end of this time the pressure with the discharge going was still .001 mm., the observations taken at the end of this time are shown by the points marked ⊙ in Curve I., Fig. 3. It is seen that while below 200 volts the deflections are reduced nearly to zero, above 300 they are unchanged. With this same anticathode a set of readings were taken with the case $-ve$; these were only half as great as with the case $+ve$, and are shown in Fig. 3 on a scale twice as large as the scale for the other readings marked by +. Relatively there is fair agreement above 300 volts. Observations taken with the case at − 6 volts do not include the photoelectrons, so that one would expect some slight difference, but we were unprepared for the great difference observed between the positive and negative currents amounting in a case observed by one of us to a ratio of 18 to 1 but variable. This when the back of the discharge tube was of gauze, and a brass electrode P was used. Finally when a small window was used, and a wire as collecting electrode the difference was reduced to one fifth of the whole. This would indicate that the difference was due to ions produced by the photoelectric electrons. Working with the case negative had the advantage of avoiding a possible error due to light in the tube.

More recently the second method of testing the effect of gas in the tube has been tried. Arrangements were made to wash out the apparatus thoroughly with hydrogen. As hydrogen is not believed to furnish any radiation of wave-length lower than 900 Ångstroms, it was supposed that this process might alter completely the X-radiation obtained. A fine capillary tube connected the discharge tube to a flask of about 1.5 liters capacity filled with electrolytically prepared hydrogen dried by passing over P_2O_5. No charcoal tube nor cooling was used. After the apparatus was pumped out, some hydrogen was admitted and pumped out, then more let in until the pressure was 19 mm. at which pressure the cock D was closed, and then the discharge tube pumped out and the admission of hydrogen controlled so that a pressure of about .003 mm. was recorded. A set of observations at 600, 500, 400, 300 volts gave values about 1/4 that with air. Cock D was then opened, the apparatus exhausted, and hydrogen introduced to 30 mm. pressure, the cock D closed, and observations repeated with a pressure .004 mm. in the discharge tube. This gave values differing only slightly from the previous ones. D was then opened, the apparatus exhausted, and hydrogen admitted to 3 mm., D closed, readings at .008 mm. gave values about 1/4 the previous ones, at .002 mm. about 1/3. 30 mm. pressure tried

again in *A* restored the higher value, 17 mm. repeated gave values about 5 per cent. higher than the first. With *D* open, hydrogen at 30 mm. was left in the apparatus over night. The next day after pumping *D* was closed at 7 mm. pressure, and a stream of hydrogen passed through the discharge tube at pressures from .005 mm. to .2 mm. the discharge going all the while with currents from .3 m.a. up. Only .2 m.a. was being used for measurements. The electrometer currents were the same afterwards as before. These various observations made it seem probable that the diminution observed as compared with air was due partially to the somewhat higher pressure in the discharge tube, but more to the use of hydrogen in the ionization chamber. Hence another test was made by pumping out and letting in air to a pressure of 5 mm., closing *D*, exhausting, and taking a reading. Hydrogen was then pumped through for two hours, and for $1\frac{3}{4}$ hour the discharge passed. For the last half hour large currents were used at a pressure of .13 mm. At the end of this time the flow of hydrogen was stopped, and readings taken. The first was a little low, probably the pressure was still high. The second was somewhat higher than it had been at the beginning, but not sufficiently so as to suggest a real difference.

Air was then let into the main apparatus and the anticathode which had been brass dusted over with graphite was removed, cleaned, polished and replaced. The flask of hydrogen was pumped out and refilled, and the experiments repeated. Hydrogen was left standing in the apparatus two hours, and for two hours the discharge was passed with hydrogen flowing through, with the same result as before. During this run it was found that at a pressure of .024 mm. and 2 m.a. current one had the same electrometer currents for 400 and 300 volts as for 0.1 m.a. at a pressure of .0016 mm. This is just the sort of change found with pressure conditions such that only a small portion of the rays striking the anticathode have the full velocity. These two sets of experiments make it rather sure that the X-radiation under observation is from the solid anticathode rather than from a gas.

In the earlier experiments no constancy had been found in the ionization currents produced by the radiation on different days, nor any relationship between the ionization and the pressure of the air. This was ascribed to two causes, the lack of a saturation current, and the presence of vapors in the air due to the use of a charcoal tube. Similar variations were found in the course of these experiments, but as this last year no charcoal tube was used, and no vapor introduced for other reasons, more uniform currents were obtained. The result of a trial with air at different pressures is shown in Table I. The air was let in

through a $CaCl_2$ drying tube and the pump, and was room air from near a window. The figures give electrometer deflections per minute for

TABLE I.

(Case − 6 volts.)

P. D.	2 Mm.	7.5 Mm.	15 Mm.	29 Mm.
600	90 (100)	126 (100)	133 (100)	126 (100)
500	43 (47.7)	62.7 (49.5)	65 (48.9)	63.2 (49.7)
400	15.4 (17.2)	23.2 (18.4)	22.5 (16.9)	24? (19.2)
300	2.1 (2.4)	2.8 (2.2)	2.6 (2.0)	2.1 (1.7)

constant cathode ray current at the different pressures. There is seen a decided increase from 2 mm. to 7.5 mm., a slight one up to 15 mm. and beyond what appears like a slight decrease. Increasing the potential on the case to − 12 volts did not alter the current. Nevertheless it is probable that at the higher pressure a saturation current was not really obtained. The relative ionization at the different voltages is shown by the figures in brackets, and does not vary markedly. The ionization current increased proportionately with the cathode ray current over a wide range tested from .007 to .3 milliampere.

Table II. shows in extended form the relative values of the ionization currents for different potential differences on the discharge tube under a variety of circumstances. The headings of the columns give the anticathode used, date, gas pressure in the chamber *A* when known. At the foot of each column is given the cathode ray current in 10^{-4} amperes, and the electrometer deflection per minute at 600 volts. The measurements are shown divided into four groups. The first group includes those where the voltages were measured electrostatically, in the others it is estimated that error if present would be in the direction of making the lower voltage readings relatively too low. In the groups after the first a number of the values have been interpolated between adjacent readings, indicated by brackets about the number at the head of the column. The second group contains two sets of measurements with an anticathode that had been soaked in a strong solution of lithium chloride. The first measurements given were made six months after the salt had been put on, and after the anticathode had been used considerably. The plate electrode *P* used in the first part of the first group, the second group and the first four sets of the third group was of aluminum, in the last five sets given it was of brass. In these five sets also the back of the discharge tube was of brass gauze, instead of a solid brass piece with a central opening. The fourth group are three sets of observations made

Table II.

Voltage.	Plate Electrode.		Wire Electrode.												Larger Window.						
	Brass Aug. 7, Air 7 Mm.	Brass Aug. 12, Air 15 Mm.	Brass Aug. 18, Air 12 Mm.	Brass Aug. 19, Air 32 Mm.	Graphite Aug. 19, Air 14 Mm.	Graphite Aug. 20, Air 18 Mm.	Graphite-H_2 Aug. 22, H_2 19 Mm.	Aluminum-H_2 Aug. 27, Air 16 Mm.	LiCl Jan., '19, Air 4 Mm.	LiCl Aug., '19, Air 9 Mm.	Platinum Aug. 1, '18, Air 4 Mm. Case + ve.	Platinum Aug. 3, '18, Air 6 Mm.	Pencilling on Brass Aug. 6, '18.	Pencilling on Brass Aug. 7, '18. Case + ve.	Brass July, '18, CS_2, Air.	Brass July, '18, CS_2, Air.	Brass May, '17, Case + ve Air.	Brass May, '17, Air.	Brass May, '17, New Window.	Cu and Pt, '14, Vacuum (d).	$d \times \frac{\text{Voltage}}{600}$.
600.....	100	100	100	100	100	100	100	100	(100)	(100)	(100)	(100)	(100)	(100)			(100)	(100)		100	100
550.....					73	69.7		70.2	67	57	63		78	61				65	(65)	76	69.8
500.....	43	48.9	52	46	55.8	50.3	49.4	48.2	35	22		42.5	52.3	42	(45)	(45)	44.5	42.8	42.5	52	41.6
450.....					38.7	34.3		30.1	18.5	9.4	23.8	23.2	32.5		27	25	27	24	24	34	25.5
430.....					37	28.7									19	21	23	20	20		
413.....					27.7	24.5				4.4					16	19	19	15	17		
400.....	12.3	16.9	19.9	19.2	23	21.1	19	18.6			13.4	14.4	20.3		14	15	15	13	14	22.2	14.8
390.....	9.3				21.8	18.6					10.7				12.7	12.4					
370.....	6.0				15.4	13			4.2											14.3	8.8
350.....	4.5		9.2		8.9	8.7		8.4		1.0	7.6		12.4		6.4	5.5				10.8	6.3
330.....			5.5		6.8	5.6						2.2		2.9							
320.....	2.9		2.85			4.7	2.9	2.9					2.4	1.7			1.4	3.8	5.2		
300.....	1	1.98	1.69		3.4	1.68	1.5	1.1	.0				1.2		2		.7			5.6	2.8
280.....			.37			0.51	.7	.7		.0		.55	.5				.4	2.3			
260.....			.18			.19		.3													
220.....	.3		.0		.0		.0	.2							0.3		.1	.1		.9	.3
200.....								.2		.0	.39	.03		.3	0.1			.0		< .2	.1
..	4	<3	1.1	0.8	0.8	1.1	1.8	1.3	3	4	3	2	3	2.9	2	2.3	30	30	13		
r																					
e.	75	133	331	204	234	385	76	312	40	68	308	152	980	618	2,440	1,860	7,200	330	830		

by Miss Barton with two windows of different thickness and size. Finally is given the relative measurements on the photoelectric current produced by the radiation at high vacuum, and the result obtained by multiplying these numbers by the corresponding difference of potential divided by 600.

If the radiation were a characteristic radiation of definite wavelengths, only the intensity of the lines increasing with the voltage, then the ionization current would be proportional to the photoelectric current. On the other hand if it were purely a general radiation in which the wavelength varies inversely with the voltage of the cathode rays, and if the energy of the ejected photoelectrons were proportional to this voltage, and capable of producing ions in numbers roughly proportional to the energy, then the relative ionization would approach the numbers in the last column. A first glance at the table is not encouraging for a definite decision, as the numbers in the rows differ even for the same anticathode. The first given for a brass anticathode, those for platinum, for brass with some carbon bisulphide, and Miss Barton's observations agree approximately between 600 and 320 volts with the last column. The others for brass, and the one for aluminum lie in between.

The only known difference between the first set of observations given for brass and the others of the first group, in addition to the difference in pressure as shown, is that in the first set the cathode ray bundle was diffuse, in the others it was a definite pencil appearing to come from one point of the cathode. In each case when graphite was used, in the one instance as powder dusted on, in the other as pencil marks, the first set of readings gave values markedly larger between 350 and 550 volts than these given by the other anticathodes, or by the graphite later. This suggested a change in the anticathode when used, but gives some grounds for supposing that one had here the K characteristic radiation of carbon which might be expected in this neighborhood. The actual deflections with the lithium chloride anticathode were smaller than with the other materials, and fell more rapidly with decreasing voltage. The surface was rough, and there was possibly a local higher pressure, so that it is not felt that this indicates surely a real difference. Below 300 volts it is safe to say that the ionization is relatively smaller than that computed by either method from the vacuum phenomenon. The ionizing power must be diminishing either as a result of diminished velocity of the ejected electrons produced by it, or of diminishing power to ionize the air directly. In the previous experiments one set of observations was recorded where there was an enormous increase in the radiation at about 400 volts. This happened after liquid air had been

removed from a charcoal tube, evidently releasing various vapors. In the hope of producing a similar effect, the vapors of ether, carbon-bisulphide and turpentine, were introduced into the apparatus, but there was no marked result. Below 200 volts especial search was made for a radiation from the lithium chloride anticathode as a characteristic X-radiation from lithium might be expected. In the first trial, not shown in the table since the cathode ray current was inconstant, effects were found down to 100 volts. This could not be repeated with the case $-$ *ve*. however. With the case $+$ *ve* an effect was found which increased with diminishing voltage down to about 170 volts and then slowly diminished. A similar effect was found however with a brass anticathode, and, as indicated in Fig. 3, with a platinum anticathode before being glowed. It evidently did not represent a radiation capable of producing ionization as there was no effect with air in the chamber when the case was negative,

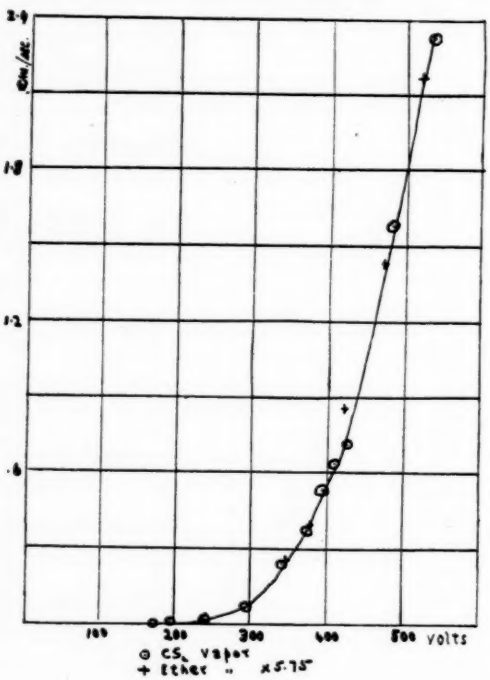

Fig. 4.

nor was it a radiation from the solid anticathode since it was removed by the ridding of the anticathode of gas in the case of the platinum. Occasionally as low as 190 volts electrometer deflections slightly greater than the natural leak were obtained, hardly enough to demonstrate surely an X-radiation from the solid anticathode. The electrometer was

however not pushed to its greatest sensitiveness. Examining the results it was noted that in the case of aluminum larger relative values at 200 and 220 volts were obtained than in any other set of this most recent group, and larger than in any other instance with the case $-ve$. Unfortunately this was not repeated nor was an attempt made to continue below 200. The observation is of interest as aluminum would supposedly have its L radiation at this point, but this was not in mind at the time. Some sets of observations are shown graphically in the curves of Figs. 4, 5, 6.

There seem to be three possibilities of explanation for the fact that

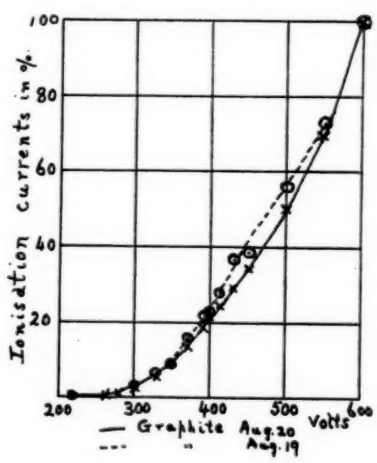

Fig. 5.

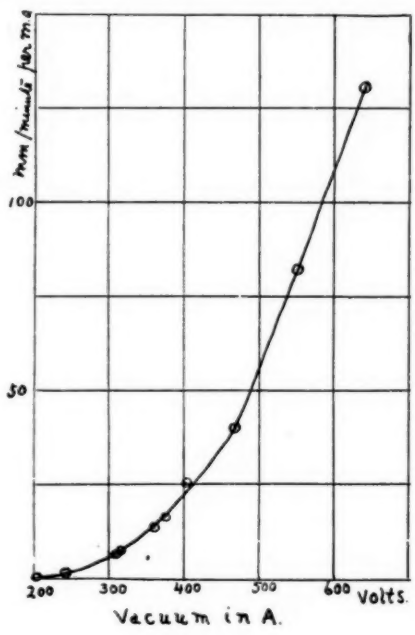

Fig. 6.

varying the anticathode changes the radiation produced so slightly. First all the anticathodes may give off so many radiations in this region that the method fails to distinguish between them, second they may all become sputtered from the cathode in the same way, or third the radiation may be largely a general radiation. This latter is probably true in any case as seen from Table II. The common classification of spectra ascribes bright line spectra to gases and vapors, in the case of the ordinary X-rays the wave-length is so small that the distances between atoms is larger than the wave-length, or a solid is relatively a thing with holes in like a gas, but in the case of these soft X-rays the wave-length is greater than the atomic distances, so that one might well expect the

relative intensity of lines to diminish unless one could produce a layer of vapor close to the anticathode. As further experiments with this particular method are being halted in the intention of trying out new methods of attack, it was thought best to publish these results as they are. A closer examination of the variation of the radiation with shorter steps in the potential difference might still bring more accurate results, with greater means at hand of keeping the heating current for the cathode constant over a long range of time.

MOUNT HOLYOKE COLLEGE,
SEPTEMBER, 1919.

SYMMETRICAL SUBDIVISION OF ANODE GLOW IN HELIUM DISCHARGE TUBES.

BY G. M. J. MACKAY.

SYNOPSIS.

Symmetrical Luminous Patterns formed on Anode by Electrical Discharge through Helium. Description of Experiment.—When a continuous electrical discharge passes through *helium* at a *pressure* of 2 cm., the *anode glow* may break up into a number of *spots* more or less regularly distributed. The most striking arrangement is the three ring type with one spot at the center of a regular pentagon, which in turn is surrounded by eleven other spots uniformly distributed about the circumference of the circular anode, which is 2.7 cm. in diameter.

WHILE investigating the characteristics of the electrical discharge in helium at different pressures, a rather peculiar phenomenon was noticed at the surface of the anode. Luminous dots, often symmetrically arranged and very sharply defined, appeared on the face of this electrode, under various conditions of pressure, current density,

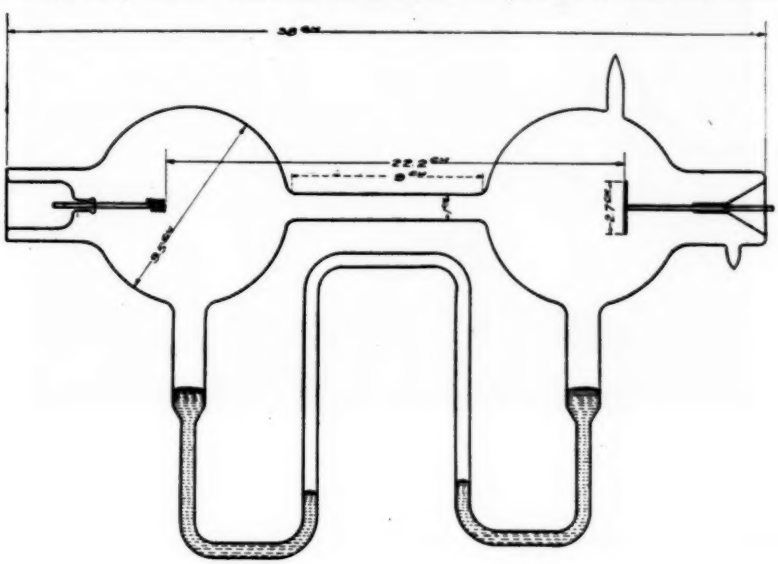

Fig. 1.

and shape of apparatus. The most regular grouping, however, was shown in the device illustrated in Fig. 1, which was designed for another purpose. This is a tube of Pyrex glass fitted with mercury manometers

and equipped with a tungsten filament cathode, and a circular anode of sheet tungsten 2.7 cm. in diameter.

After a good exhaust treatment, helium, obtained from Mr. P. E. Haynes, of the Linde Air Products Company, was admitted to the bulb through a tube containing charcoal immersed in liquid air, and the apparatus sealed off at a pressure of 2 cm.

The discharge was produced by a 1,500-volt direct current generator using a Kenotron with different filament temperatures as a regulating resistance. The cathode of the helium tube was not separately excited, the voltage being sufficient to start the glow with the cold filament. As the current was changed, various arrangements of spots appeared on the anode as shown by the photographs in Fig. 2, taken at an angle

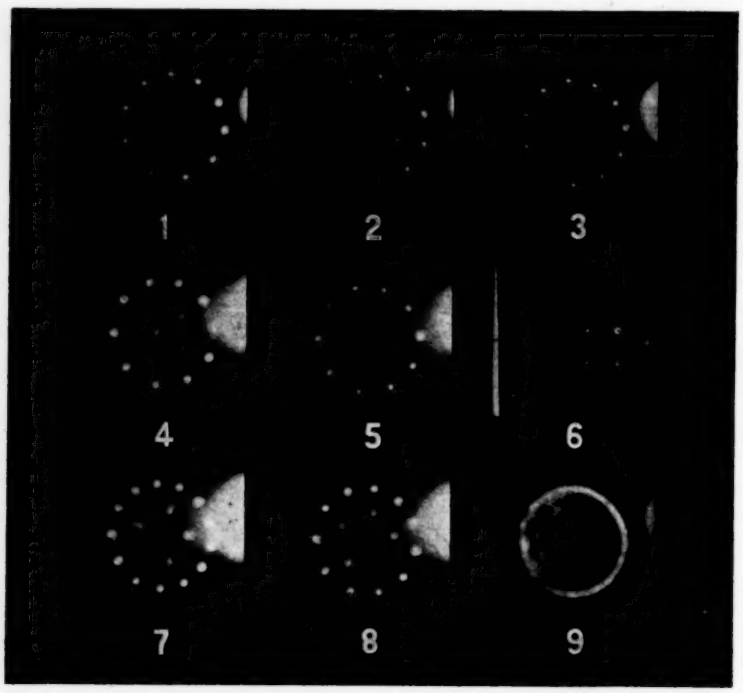

Fig. 2.

of about 30° to the axis of the tube.[1] In the order of the number of the dots, the volt-ampere characteristics were as follows:

(1) 420 volts .023 amp.	(4) 440 volts .016 amp.	(7) 440 volts .025 amp.
(2) 460 .012	(5) 435 .018	(8) 440 .020
(3) 430 .011	(6) 440	(9) 430 .085

[1] The four inner spots of No. 6 have been intensified since otherwise they would be hardly perceptible.

The first two and last two were apparently the most stable combinations, the others occurring only momentarily. It was quite easy, however, to reproduce the series from time to time. At the higher currents, as shown in the last picture, the separate spots merged into what appeared to be a rotating ring.

The spectrum of the glow showed mercury to be present as well as helium, but in other types of apparatus where mercury was excluded, phenomena of like nature were observed.

The arrangement of these spots is interesting on account of their general similarity to the configurations shown by Mayer's magnets used by Sir J. J. Thompson as an analogy in his conception of atomic structure.

Somewhat similar figures have been obtained in air at low pressures (.0144 mm.) by O. Lehmann[1] but of not so elaborate pattern.

Even at atmospheric pressure, with helium, there is a tendency for the anode glow to break up into a number of separate dots, though the current may be an ampere or more, and the arc gap less than a centimeter. This, in connection with other phenomena, makes it appear that helium at comparatively high pressures shows characteristics which normally occur in other gases only at relatively low pressures.

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[1] Ann. d. Physik, 7, 8, 1902.

NOTE ON THE RETARDED POTENTIALS.

BY MAX MASON.

SYNOPSIS.

Electromagnetic Field Equations, Mathematical Theory.—This note gives a *simple method of deriving the electromagnetic scalar and vector potentials.* From the field equations it follows in the well-known manner that these potentials satisfy the non-homogeneous form of the wave equation. This equation is reduced, in Poisson's manner, to one of two independent variables r, t by introducing a function U which is the integral of the potential over the surface of a sphere of radius r about the point at which the potential is sought, divided by r. The potential is given by the limiting value of U divided by r as r approaches zero. In the method of Abraham the equation is solved for U and the potential determined from U as indicated. It is, however, not necessary to determine U itself, but only the value of its derivative with respect to r for $r = 0$, since this derivative gives the limiting value. It follows, after a change of independent variables in the equation for U, that a single integration determines the potential in a simple and direct manner.

STATEMENT OF THE PROBLEM AND APPLICATION OF THE METHOD OF POISSON.

THE following method of derivation of the retarded potentials seems of considerably greater simplicity than those previously employed. For completeness and clarity the introduction of the potentials and the reduction of the equations for the potentials by the method of Poisson, although well known, are included.

It is required to determine the electric and magnetic vectors $\boldsymbol{E}$ and $\boldsymbol{B}$ through the equations

$$\text{Curl } \boldsymbol{B} = \frac{1}{c}\dot{\boldsymbol{E}} + \frac{1}{c}\rho\boldsymbol{v}, \tag{1}$$

$$\text{Curl } \boldsymbol{E} = -\frac{1}{c}\dot{\boldsymbol{B}}, \tag{2}$$

$$\text{div } \boldsymbol{B} = 0, \tag{3}$$

$$\text{div } \boldsymbol{E} = \rho. \tag{4}$$

It will be assumed that the charges of density ρ, whose field is to be determined, were at rest in a finite region of space until after the time t_0, the field having been electrostatic up to that time, and that the equation of conservation of charge,

$$\text{div } \rho\boldsymbol{v} + \dot{\rho} = 0, \tag{5}$$

is satisfied at all times.

Equation (3) indicates the assumption of a vector potential $\boldsymbol{A}$,

$$\boldsymbol{B} = \operatorname{curl} \boldsymbol{A}.$$

It follows from (2) that a scalar potential φ must exist, such that

$$\boldsymbol{E} = -\nabla\varphi - \frac{1}{c}\dot{\boldsymbol{A}}.$$

Since this field is assumed to be electrostatic until after the time t_0, the following initial conditions on φ and $\boldsymbol{A}$ must be satisfied:

$$(6)\qquad \begin{aligned} (\varphi)_{t \leqq t_0} &= \varphi_0, \qquad \left(\frac{\partial\varphi}{\partial t}\right)_{t \leqq t_0} = 0, \\ (\boldsymbol{A})_{t \leqq t_0} &= 0, \qquad \left(\frac{\partial \boldsymbol{A}}{\partial t}\right)_{t \leqq t_0} = 0. \end{aligned}$$

If the expressions for $\boldsymbol{B}$ and $\boldsymbol{E}$ are introduced into equations (1) and (4) each of the resulting equations involves both φ and $\boldsymbol{A}$, viz.:

$$(1')\qquad -\nabla^2\boldsymbol{A} + \nabla \operatorname{div} \boldsymbol{A} = -\frac{1}{c}\nabla\varphi - \frac{1}{c^2}\ddot{\boldsymbol{A}} + \frac{1}{c}\rho\boldsymbol{v},$$

$$(4')\qquad -\nabla^2\varphi - \frac{1}{c}\operatorname{div}\dot{\boldsymbol{A}} = \rho.$$

It is clear that if φ and $\boldsymbol{A}$ be any potentials giving a field $\boldsymbol{E}$, $\boldsymbol{B}$ then $\boldsymbol{A}' = \boldsymbol{A} + \Delta\psi$, $\varphi' = \varphi - (1/c)\dot{\psi}$ will give the same field, whatever be ψ. It is necessary to add a further condition on φ and $\boldsymbol{A}$ to uniquely determine them, and it is natural to assume such a one as will simplify the problem by separating the potentials in equations (1') and (4'). Such a condition is

$$(7)\qquad \operatorname{div}\boldsymbol{A} + \frac{1}{c}\dot{\varphi} = 0.$$

The equations then become

$$(1'')\qquad \nabla^2\boldsymbol{A} - \frac{1}{c^2}\ddot{\boldsymbol{A}} = -\frac{1}{c}\rho\boldsymbol{v},$$

$$(4'')\qquad \nabla^2\varphi - \frac{1}{c^2}\ddot{\varphi} = -\rho.$$

Conversely, if solutions of these equations are obtained, the resulting vectors $\boldsymbol{E}$ and $\boldsymbol{B}$ will satisfy the field equations (1)–(4) provided that $\boldsymbol{A}$ and φ satisfy (7). Since the equation (5) of conservation of charge is assumed to hold, it follows from (1'') and (4'') that the function

$$W = \operatorname{div}\boldsymbol{A} + \frac{1}{c}\dot{\varphi},$$

formed for any solutions of (1″) and (4″), satisfies the equation

$$\nabla^2 W - \ddot{W} = 0.$$

This function must also, on account of equations (6) satisfy the initial conditions

$$(W)_{t \leq t_0} = \left(\frac{\partial W}{\partial t}\right)_{t \leq t_0} = 0.$$

It will be shown later that these conditions force W to be identically zero, so that the solutions φ and $\boldsymbol{A}$ of equations (1″) and (4″) and the initial conditions (6) also satisfy the equation of connection (7). The vectors determined by them therefore satisfy the field equations.

The problem of the determination of φ and $\boldsymbol{A}$ being analytically similar, attention will be directed to φ. The problem[1] may be reduced to one of two independent variables in the manner of Poisson, by the introduction of a function U defined as

$$U(r, t) = \frac{1}{r}\int_{(r)} \varphi d\sigma, \tag{8}$$

where the integration is taken over a sphere of radius r about the point P at which the potential φ is to be determined, $d\sigma = r^2 d\omega$ being the element of area. If U be found, the value of φ at the point in question is given by

$$4\pi\varphi = \lim_{r=0} \frac{U(r, t)}{r} = U_1(0, t), \tag{9}$$

where partial differentiation of the function $U(r, t)$ with respect to its first argument is denoted by the subscript 1.

The function U must thus vanish for $r = 0$,

$$U(0, t) = 0, \tag{10}$$

and from (6) it must satisfy the conditions

$$U(r, t_0) = \int_{(r)} \frac{\varphi_0}{r} d\sigma, \tag{11}$$

$$U_2(r, t_0) = 0, \tag{12}$$

where the subscript 2 denotes differentiation of U with respect to its second argument. From the definition of U and the equation (4″) it is shown without difficulty that U must satisfy the differential equation

$$U_{11}(r, t) - \frac{1}{c^2} U_{22}(r, t) = -\int_{(r)} \frac{\rho}{r} d\sigma. \tag{13}$$

Abraham has applied a method of Riemann for the determination

[1] M. Abraham, Acc. dei Lincei (5), 14¹, p. 7, 1905, and Theorie der Elektrizität, Vol. II., p. 39.

of U and thus of φ. His analysis is however unnecessarily complicated, for it is not necessary to find $U(r, t)$ itself, since, according to equation (9), the potential φ is determined by $U_1(0, t)$, *i.e.*, by the value of the derivative of U with respect to r at $r = 0$.

Solution by Change of Variable.

The differential equation (13), holding for all values of its arguments, remains an identity in r and t when $t - r/c$ is substituted for t, *i.e.*,

$$U_{11}(r, t - r/c) - \frac{1}{c^2} U_{22}(r, t - r/c) = -\int_{(r)} \frac{\{\rho\}}{r} d\sigma,$$

where

$$\{\rho\} = \rho(x, y, z, t - r/c).$$

Now

$$\frac{\partial}{\partial r}\left[U_1(r, t - r/c) + \frac{1}{c} U_2(r, t - r/c) \right] = U_{11} - \frac{1}{c} U_{12} + \frac{1}{c} U_{21} - \frac{1}{c^2} U_{22}$$

$$= U_{11} - \frac{1}{c^2} U_{22}.$$

Consequently integration of the above equation with respect to r from $r = 0$ to $r = c(t - t_0)$ gives

$$\left[U_1(r, t - r/c) + \frac{1}{c} U_2(r, t - r/c) \right]_{r=0}^{r=c(t-t_0)} = -\int_0^{c(t-t_0)} \int_r \frac{\{\rho\}}{r} d\sigma dr,$$

or

$$U_1(c(t - t_0), t_0) - U_1(0, t) + \frac{1}{c} U_2(c(t - t_0), t_0) - \frac{1}{c} U_2(0, t) = -\int_{c(t-t_0)} \frac{\{\rho\}}{r} d\tau, \tag{14}$$

the integral on the right being taken through the volume of a sphere of radius $c(t - t_0)$.

From equation (11), which holds identically in r, it follows on differentiating with respect to r and substituting $c(t - t_0)$ for r, that, writing $d\sigma = r^2 d\omega$,

$$U_1(c(t - t_0), t_0) = \left[\frac{\partial}{\partial r} \int_{(r)} \varphi_0 r d\omega \right]_{r=c(t-t_0)} = \int_{c(t-t_0)} \left(r \frac{\partial \varphi_0}{\partial r} + \varphi_0 \right) d\omega.$$

The third and fourth terms of the left member of equation (14) vanish on account of the initial conditions (12) and (10) respectively, so that, using (9),

$$4\pi\varphi = \int_{c(t-t_0)} \frac{\{\rho\}}{r} d\tau + \int_{c(t-t_0)} \left(r \frac{\partial \varphi_0}{\partial r} + \varphi_0 \right) d\omega.$$

It has been assumed that the initial potential φ_0 due to the charges in

question was, until after the time t_0, the electrostatic potential due to the charges, which were confined to a finite volume of space. This potential is consequently "regular at infinity," so that, however large r becomes,

$$\left| r\frac{\partial \varphi_0}{\partial r} + \varphi_0 \right| < \frac{f}{r},$$

where f is a finite number. If then the initial time t_0 be allowed to approach $-\infty$, the equation results,

$$\varphi = \frac{1}{4\pi}\int \frac{\{\rho\}}{r} d\tau,$$

the integral being taken over all of space.

In the same manner the equation

$$\boldsymbol{A} = \frac{1}{4\pi c}\int \frac{\{\rho \boldsymbol{v}\}}{r} d\tau$$

is obtained.

It follows as a result of the unique integral representation that the function

$$W = \operatorname{div} \boldsymbol{A} + \frac{1}{c}\dot{\varphi}$$

mentioned above is identically zero, since it satisfies the homogeneous equation

$$\Delta^2 W - \frac{1}{c^2}\ddot{W} = 0$$

and the initial conditions

$$(W)_{t \leqq t_0} = \left(\frac{\partial W}{\partial t}\right)_{t \leqq t_0} = 0,$$

so that the required equation of connection is satisfied between the potentials φ and $\boldsymbol{A}$.

THE UNIVERSITY OF WISCONSIN,
MADISON, WISCONSIN.

NOTE ON PAPER BY C. V. KENT ON THE OPTICAL CONSTANTS OF LIQUID ALLOYS.

BY H. A. WILSON.

THE results described by Mr. C. V. Kent in the PHYSICAL REVIEW for December, 1919, form an important confirmation of the electron theory of metallic conductivity in the case of liquid metals and alloys. Some of his conclusions, however, appear to be based on a misconception with respect to the electron theory. According to Drude's theory as used by Mr. Kent the conductivity for zero frequency is equal to Ne^2/mW and the conductivity for frequency $\omega/2\pi$ is equal to $Ne^2W/m(\omega^2 + W^2)$. Thus the conductivity for high frequencies is much less than for zero frequency. Mr. Kent calculates Ne^2/mW from his optical results and finds it equal to the electrically determined conductivity for zero frequency. This confirms the theory and shows that the conductivity diminishes as the frequency increases in the way indicated by the theoretical expression. But Mr. Kent apparently considers that his results show that the conductivity does not vary with the frequency. He therefore remarks that the various theories which show a diminution of the conductivity at high frequencies are erroneous. But Drude's theory shows the same diminution as the later theories so that Mr. Kent ought to have concluded that Drude's theory is equally erroneous. But he uses Drude's theory and finds that his results confirm it. Mr. Kent did not calculate the conductivity for the high frequencies but only that for zero frequency. If he had calculated the conductivity for high frequencies from his results he would have found it much smaller than that for zero frequency.

As to the later theories of "Livens and others" it should be noted that they do not differ essentially from Drude's theory as used by Mr. Kent. They are merely developments of Drude's theory in which the resistance to the motion of electrons is estimated on the kinetic theory and the distribution of velocities among the electrons allowed for. Mr. Kent's results therefore confirm the theories of "Livens and others."

RICE INSTITUTE,
HOUSTON, TEXAS,
February 5, 1920.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE NEW YORK MEETING, FEBRUARY 28, 1920.

THE 102d meeting of the American Physical Society was held in room 301, Fayerweather Hall, Columbia University, New York, on Saturday, February 28, 1920. There was a single session beginning at 10 o'clock A.M., with an attendance of about one hundred and fifty. The following program of seventeen papers was presented, four being read by title only:

Improved Form of McLeod Gauge. AUSTIN BAILEY. (Read by title.)

The Polarization of Radiation by Diffraction Gratings. L. R. INGERSOLL. (Read by title.)

Electron Currents in Some Non-Metallic Vapors. F. L. MOHLER AND PAUL D. FOOTE.

Atomic Theory and Low Voltage Arcs in Cæsium Vapor. PAUL D. FOOTE AND W. F. MEGGERS.

Note on Visibility of Radiation and Spectral Energy Measurements. W. W. COBLENTZ. (Read by title.)

The Spectophotoelectric Sensitivity of Molybdenite as a Function of the Direction of the Current Through the Crystal. W. W. COBLENTZ. (Read by title.)

On the K-Series of X-Rays. WILLIAM DUANE AND WILHELM STENSTRÖM.

On the X-Ray Spectrum of Tungsten. WILLIAM DUANE AND R. A. PATTERSON.

The Emission of Electrons from Oxide Coated Filaments Under Positive Bombardment. C. DAVISSON AND L. H. GERMER.

Crystal Analysis of Metallic Oxides. WHEELER P. DAVEY AND E. O. HOFFMAN.

Note Concerning the Possible Non-Radiality of the Observed Deflections of Light Rays During the Solar Eclipse of May 29, 1919. LOUIS A. BAUER.

The Einstein Deflection for High Speed Material Particles. LEIGH PAGE.

How a Reed Instrument is Played in Tune. JOHN B. TAYLOR.

Further Developments Relative to the New Singing Tube. CHARLES T. KNIPP.

The Distribution of Energy About a Point Source. PAUL R. HEYL.

On Electrostatic Transformers and Coupling Coefficients. F. C. BLAKE.

The Selective Reflection of Heat Waves by Linear Resonators. E. C. WENTE.

Upon the completion of the program there was a general discussion, introduced by I. G. Priest, of the desirability of distributing abstracts of papers in advance of the oral presentation. The matter was referred to the Council for further consideration.

At a meeting of the Council held at 8:30 o'clock P.M., on February 28, 1920, the immediate publication of a printed directory of members was authorized. The desirability of general index, covering fifty volumes of the PHYSICAL REVIEW, was considered, and several other important items of business were introduced, to be reported to the Society later. The following elections were made: *elected to associate membership:* A. S. Coolidge, Richard Hamer, John Hays Hammond, Jr., R. M. Holmes, LeRoy Archibald McColl, S. Stuart Mackeown, Joseph Chrisman MacKinnon, Andrew L. Matte, A. I. Peterson, Louis G. Pooler: *elected to regular membership:* Frank L. West; *transferred from associate to regular membership:* H. Clyde Snook.

DAYTON C. MILLER, *Secretary.*

IMPROVED FORM OF MCLEOD GAUGE.[1]

BY AUSTIN BAILEY.

ANY form of McLeod gauge is open to objections on the basis of the accuracy of the measured pressure, but since it has been almost universally adopted as a quick means of obtaining an indication of the pressure, any improvement in its form which makes it more easily operated is considered of real value.

The accompanying diagram shows an improved form of McLeod gauge. At S is a three-way stopcock which connects the mercury reservoir R either to the tube P, which is drawn into a fine point and open to the air, or to the tube A, which is connected to a water aspirator. At T is the usual air trap to collect any possible entrained air from R, but with this type of gauge such possibility is small. At G is a ground glass joint with a mercury seal which connects the compression chamber V to the rest of the gauge. M is simply a convenient comparison manometer.

The stopcock S should normally be turned so as to connect R to the aspirator through A. In this position V will be open to the pressure which we desire to measure. To make a reading the stop-cock S is turned so as to open R to the atmospheric pressure through P. When the mercury in the comparison capillary has risen to a point level with the top of the capillary on V, the reservoir is closed by the stopcock at S and the reading is taken.

The advantages of this gauge are as follows: (1) The entire gauge stands on the table and the scale at C is at a convenient height to be easily read. (2) There is no possibility of spilling mercury or of having it rise too quickly into

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

the compression chamber, since the rate of flow is governed by the size of the fine point at P and the rate decreases as the mercury rises in V. (3) The height from the lowest mercury level to the outlet tube being greater than a barometric height no mercury can run over into the apparatus that is connected to the gauge. The same is true on the side connected to the aspirator. (4) The apparatus has no rubber connections to contaminate the mercury. (5) The gauge can be used over a wide range of pressure by simply removing

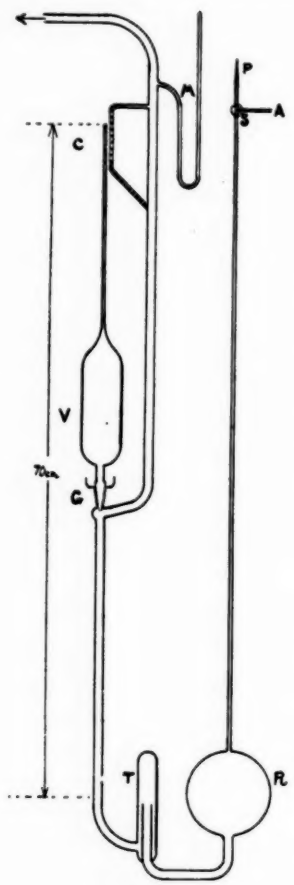


Fig. 1.

the compression chamber V and replacing it by another of different volume, but having the same size capillary. (6) Each compression chamber can be removed and accurately calibrated. There is nothing indefinite about the point at which the compression chamber becomes isolated from the rest of the gauge. (7) Since the compression chamber can thus be removed and calibrated, it makes the gauge of particular value as a piece of laboratory apparatus for which the student can devise his own direct reading scale.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
January, 1920.

The Polarization of Radiation by Diffraction Gratings.[1]

By L. R. Ingersoll.

USING the same spectro-bolometric apparatus with which he has previously investigated various polarization phenomena in the early infrared spectrum, the writer has recently measured the polarizing effect of diffraction gratings for this spectral region. Both reflecting and transmission (collodion replica) gratings show in the undiffracted beam at normal incidence a very decided maximum of the polarizing effect for a wave-length equal to the grating space. Du Bois and Rubens, while they investigated the corresponding spectral region for wire gratings, did not find this maximum,—probably because the restrahlen which they used gave them too few points on the curve. The maximum which they found for a wave-length of about one tenth of the grating space apparently has no connection with this, but is to be made the subject of further investigation by the writer.

This maximum of polarizing effect is very sharp and is accompanied by secondary maxima for wave-lengths of one half and one third of the grating space. Further investigation of this phenomena shows that, for the reflecting grating at any rate, the effect is entirely due to the change in reflecting power for radiation in which the electric vector is perpendicular to the rulings of the grating. For such radiation the grating shows a decided decrease in reflecting power at a wave-length equal to the grating space, whereas for radiation of the opposite polarization there is no effect. For transmission gratings the matter seems to be somewhat more complicated.

It is of interest to note in this connection that Rayleigh has predicted and Wood has, to a certain extent, verified the fact that when a given wave-length in any higher order spectrum is just passing (tangentially) off the grating there should be a decided maximum of energy of this polarization (electric vector perpendicular to rulings) for this wave-length in the other spectra. In the light of the present result it would seem that this energy is taken from the undiffracted beam.

Electron Currents in Some Non-Metallic Vapors.[1]

By F. L. Mohler and Paul D. Foote.

THE type of tube employed for the measurement of ionization and resonance potentials in metallic vapors has been modified by the addition of a fourth electrode. The accelerating potential was applied between the cathode and the inner of two cylindrical grids at the same potential and inclosing most of the space traversed by the electrons. With a small retarding field between the outer grid and plate the variations in plate current give sharp indications of resonance potentials. Ionization was measured by noting the point at which positive current flowed from the plate when this retarding

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

field was larger than the accelerating field. Other arrangements of electrical circuits were also tried.

With this type of tube curves showing resonance and ionization potentials in phosphorus, iodine and sulphur were obtained giving the following mean results:

Phosphorus	—	Resonance	$5.80 \pm .1$	volts,	Ionization	$13.3 \pm .5$	volts.
Iodine	—	"	$2.34 \pm .2$	"	"	$10.1 \pm .5$	" .
Sulphur	—	"	$4.78 \pm .2$	"	"	$12.2 \pm .5$	" .

Points of inelastic collision of the first type, resonance, were very sharp in phosphorus but ionization points were less distinct in all three elements. A notable feature of the partial current curves was the large positive initial potential correction to be added to the ionization potential in nearly all cases especially in phosphorus where it was sometimes as much as four volts. This is undoubtedly due to the contact potential between surfaces attacked by phosphorus and the hot cathode.

Curves obtained in the vapor of yellow phosphorus at about 30° C. and red phosphorus at 400° C. gave identical results showing that the vapor is probably of the same constitution in this range. Measurements in iodine were made in the range 0° C. to 20° C. and in sulphur at about 120° C. Insufficient knowledge of the spectra of these elements makes application of the quantum relation $Ve = h\nu$ of doubtful value. The resonance potentials indicate the existence of single-line spectra as follows: Phosphorus, $\lambda = 2130$ Å.; Iodine, $\lambda = 5300$ A.; Sulphur, $\lambda = 2580$ Å.

BUREAU OF STANDARDS,
February 12, 1920.

ATOMIC THEORY AND LOW VOLTAGE ARCS IN CÆSIUM VAPOR.[1]

BY PAUL D. FOOTE AND W. F. MEGGERS.

THEORETICAL.

ON the basis of several possible theories of atomic structure it is shown that the *normal* operation of an arc below ionization might result in the excitation of a single line spectrum, a single series spectrum, or a group spectrum consisting of certain lines of different series. This latter conclusion follows from an extension of Bohr's theory. Thus if inelastic electronic-atomic impact occurred resulting in the ejection of an electron to the pth ring, the electron in returning to the nth ring or equilibrium may produce any combination of lines represented by interorbit transitions within this range, the single line spectrum being a special case where $p = n + 1$.

A simple explanation is offered of fluorescence phenomena in vapors of the alkali metals.

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

A mechanism of absorption of radiation is described and the theory proposed by K. Compton that the ionization of an atom below the ionization potential may be explained by absorption of radiation arising in other atoms from electronic-atomic impact of insufficient energy to ionize is further discussed. This hypothesis suggests that vapors of the alkali metals may be so stimulated that the 1st and 2d subordinate series lines, instead of the principal series, tend to become absorption lines.

EXPERIMENTAL.

The cæsium spectrum was photographed for various accelerating voltages from λ 3878 to λ 9208 by use of dicyanin stained plates. The sensitivity of the plates was investigated by density measurements of the spectrum of a black body having a known energy distribution. The general characteristics of the plates were determined and all lines of the cæsium spectrum were reduced to an absolute scale of intensity by means of density measurements made with a microphotometer and consideration of the plate sensitivity. No evidence of group or single series spectra could be obtained. Thus the ratio of intensities of the first and second lines of the principal series, both of which should appear in a single series spectrum or above 2.7 volts in a group spectrum, rapidly approaches infinity as the accelerating potential in the arc is decreased. This ratio is 350 in a 7 volt arc, 2100 at 4 volts, 10,500 at 3.4 volts and as near infinity as can be measured at 2.8 volts. Similarly the intensity ratio of either $1.5s - 2p_1$ or $1.5s - 2p_2$ to any other line approaches infinity at low voltage, proving for the first time the existence of a single line spectrum rather than a single series or group spectrum; in the case of cæsium the doublet λ 8521 and λ 8943.

The doublet $1.5s - 2p$ is alone produced under excitation of 1.5 to 3.9 volts accelerating field. The intensity of both of these lines gradually increases approximately proportional to the total number of electrons reaching the anode until the ionization potential is reached. At this point a pronounced decrease in intensity of these two lines occurs amounting to the factor one third.

This decrease takes place at the voltage at which the complete line spectrum is produced and is readily explainable on the basis of Bohr's theory; in fact it affords a strong argument for this theory. Thus the lines $1.5s - 2p_{1_2}$ are the result of inelastic collision with electrons having velocities between 1.45 and 3.9 volts but as the latter voltage is exceeded electrons, which at a slightly lower velocity would have given rise to $1.5 - 2p_{1_2}$, now produce the complete series spectrum; and any line of the series $1.5s - mp$ where $m > 2$ is necessarily excited at the sacrifice of $1.5s - 2p_{1_2}$.

Above a certain voltage, the intensity of any line per unit number of electrons reaching the anode attains a saturation value, in agreement with the quantum hypothesis which requires that the number of quanta radiated be proportional to the number of collisions, and hence (approximately) to the number of electrons present.

Curves are given showing the relative intensities of the prominent cæsium

lines at various voltages. The ratio of intensities of the components of the first doublet of the principal series λ 8521/λ 8943 is constant and equal to 1.5 from 1.5 volts to 120 volts.

The cæsium arc of the type employed does not rectify alternating current of 120 volts.

Sodium and potassium occurring as an impurity of the cæsium similarly exhibited the single line or doublet spectrum $1.5s - 2p_{1_2}$ below their respective ionization potentials.

Only two types of inelastic impact between electrons and atoms of the alkali metal vapors occur, at potentials known as the resonance and ionization potentials and given by the quantum relation $h\nu = eV$ where $\nu = 1.5s - 2p_1$ and $\nu = 1.5s$.

BUREAU OF STANDARDS.

NOTE ON VISIBILITY OF RADIATION AND SPECTRAL ENERGY MEASUREMENTS.[1]

BY W. W. COBLENTZ.

IN a previous communication[2] data were given on the visibility of radiation of the average eye, based upon the measurements of 125 observers, using a flicker photometer, and an illumination of 50 m.c. upon the rotating sectored disk.

These data, were subsequently revised,[3] in the red, by making a correction for the effect of diffuse light[4] upon the visibility measurements, which correction had been overlooked in assembling the data for publication.

The object of this communication is to record (Table I.) these revised data and to discuss the radiation measurements upon which the visibility work depends.

The source of the spectral radiation stimulus was a "Crescent Aero" acetylene flame, with a diaphragm opening 8 mm. high. The spectral intensities were measured at the eyepiece in the prismatic spectrum as used in the visibility work. This seems more logical than to rely solely upon the spectral energy distribution as determined by computation and a knowledge of the temperature of the source,[5] which should be used as a check upon the accuracy of the observations. There are numerous uncertainties in reducing the data from prismatic to normal energy distribution, or vice versa, and reliance upon computations does not seem sufficient for the energy evaluation. In future visibility investigations, the temperature of the source of radiation should be known, and the computations should be checked by direct observations.

Owing to an error made in reducing the original observations to a normal

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

[2] Coblentz and Emerson, PHYS. REV., (2), 9, p. 87, 1917.

[3] Bul. Bur. Stds., 14, p. 219, 1917.

[4] Coblentz, Jour. Franklin Inst., 181, p. 237, 1916.

[5] Hyde, Forsythe and Cady, Astrophys. Jour., 48, p. 71, 1918.

spectrum, recently the writer found it necessary to revise the data on the normal spectral energy distribution of the acetylene flame in the violet end of the spectrum, as fully explained elsewhere.[1] This revision does not affect the visibility work, because the observed prismatic spectral energy distribution was used in evaluating the visibility measurements. This should be kept in mind clearly when considering the accuracy of the visibility work in the region of 0.5 μ to 0.75 μ.

The revised data are given in column 2, of Table 2; also the computed (by

TABLE I.

Visibility of Radiation of a Group of 125 Observers and of Selected Group of 29 Observers having Closely the Average Visibility.

Wave-length μ = 0.001 Mm.	Visibility of Radiation.	
	125 Observers.	29 Observers.
0.493 μ	.223	.230
.502	.350	.357
.512	.553	.559
.523	.771	.778
.534	.908	.908
.546	.983	.987
.552	.998	1.000
.559	1.000	.998
.573	.952	.955
.580	.899	.900
.587	.823	.824
.596	.734	.735
.604	.636	.636
.613	.517	.517
.623	.390	.387
.633	.267	.264
.643	.165	.164
.654	.095	.093
.665	.046	.045
.678	.0202	.0197
.690	.0086	.0075
.703 μ	.0034	.0033

H. Kahler) spectral energy distribution using a color temperature[2] of 2360° K. The data are in excellent agreement[3] in the region of 0.5 μ to 0.75 μ, where the radiometric measurements could be made accurately. Beyond 0.7 μ, especially for the Bray tip (Eastman Kodak Co., burner) the observed emission is higher (the relative spectral energy is steeper, indicating a lower apparent temperature) than the computed emission.

The criticism made by Hyde and his collaborators,[4] that the writer's experi-

[1] Bur. Stds., Sci. Paper No. 362, 1920.

[2] Hyde, Forsythe, and Cady (2), 13, p. 157, 1919.

[3] Coblentz, PHYS. REV. (2), 14, p. 168, 1919.

[4] PHYS. REV. (2), 14, p. 379 (see p. 382), 1919.

mental data, in the red, do not fit the black-body energy distribution better than 7 to 8 per cent. ($\pm$ 3 to $\pm$ 4 per cent. in the region of 0.5 μ to 0.75 μ is a fairer estimate), is inconsistent with their own observations, made with a spectral pyrometer, which show that beyond 0.70 μ the obesrved emissivity of the acetylene flame is slightly greater (amounting to about 7 per cent. at 0.75 μ) than that of a black body at 2360° K. In other words, their data show, as was found by the writer[1] some years ago, that, in making a color match, the eye is not able to distinguish a difference in the spectral energy match amounting to perhaps 3 to 5 per cent. in the extreme red end (0.75 μ) of the visible spectrum. The supposed disagreement appears to be in the interpretation of the data, rather than in the experimental results themselves. For, if

TABLE II.

Spectral Energy Distribution of a Cylindrical, "Crescent Aëro," Acetylene Flame; Observed and Computed Using Wien's Eq., $C_2 = 14{,}350$ and $T = 2360°$ K.

Wave-length μ=.001 Mm.	E Observed, 1911-1916.		E Revised, 1919.		E' Computed.	$\frac{100E'}{E}$.	
0.400 μ	5.9		5.		3.3	66.	
.425	8.2		7.		5.5	79.	
.440	10.0		8.5		7.6	89.4	
.450	11.5		10.0		9.25	92.5	
.460	13.0		11.8		11.2	94.9	
.475	16.0		15.0		14.6	97.4	
.500	21.9		20.9		21.0	100.5	
.520	27.9		27.5		27.3	99.3	
.525	29.5		29.2		29.2	100.0	
.540	35.0		34.6		34.6	100.0	
.550	38.9		38.9		38.8	99.8	
.560	42.9		42.9		43.1	100.4	
.575	49.8		49.8		49.9	100.2	
.580	52.2		52.2		52.4	100.3	
.600	62.1		62.5		62.9	100.6	
.620	73.0		73.3		74.0	101.1	
.625	75.7		76.1		76.8	100.8	
.640	84.7		85.0		86.0	101.1	
.650	91.1		91.2		92.1	101.0	
.660	97.4		97.6		98.5	100.7	
.675	107.5	Bray tip	107.5	Bray tip	108.0	100.4	Bray tip
.680	110.9		110.9		111.3	100.7	
.700	124.6	125.8	124.1	125	124.1	100.0	99.3
.720	138.5	140.7	137.5	140	137.2	99.8	98.0
.725	141.9	144.5	141.0	143	140.5	99.6	97.8
.740	152.0	155.8	151.0	154	150.2	99.5	96.8
.750	158.9	163.7	157.9	163	157.2	99.5	96.5

the emissivity is higher in the red, then the ratios must decrease as shown in columns 5 and 6 of Table II.

[1] Bul. Bur. Stds., 6, p 317, 1910.

Some years ago the writer also "discovered" selective emission[1] at 0.75 μ in the acetylene flame; but after more mature consideration of the data,[2] which had been obtained with a fluorite prism having an angle of only about 39°, he concluded that the dispersion was too small to indicate a true protuberance at 0.75 μ in the spectral energy curve.[3] The absorptivity curve of the acetylene flame[4] undergoes a rather rapid change in curvature, in this region of the spectrum, beyond which point it becomes more uniform. This would affect the smoothness of the spectral energy curve, making it steeper beyond 0.8 μ and indicating a lower (gray-body) temperature.

From the absorptivity data, and from the position of the maximum emission, it appears that beyond 0.8 μ the spectral energy distribution is closely that of a gray-body[5] at a somewhat (perhaps 100°) lower temperature. From a consideration of the absorptivity data, this is to be expected. From the absorptivity data in the visible spectrum it seems that one should not expect exact coincidence between the observed spectral energy curve of the acetylene flame and that computed from the color temperature.

WASHINGTON, D. C.,
January 20, 1920.

THE SPECTROPHOTOELECTRIC SENSITIVITY OF MOLYBDENITE AS A FUNCTION OF THE DIRECTION OF THE CURRENT THROUGH THE CRYSTAL.[6]

BY W. W. COBLENTZ.[7]

IN a previous paper[8] it was shown that the electrical conductivity of molybdenite, in the form of long narrow ribbons, soldered to copper electrodes, depends greatly upon the direction of the current through the crystal. Merely reversing the direction of the current through the crystal changed the conductivity by 1 to 70 per cent., depending upon the crystal.

In a subsequent communication,[9] it was shown that in a certain sample of molybdenite, for the visible spectrum extending to about $\lambda = 0.647\ \mu$, the electrical response is photo-positive or photo-negative, depending upon the voltage applied to the terminals.

The object of the present communication is to call attention to recent observations which show that the photoelectrical response depends not only upon the magnitude of potential applied to the terminals of the molybdenite strip but also upon direction of the (dark) current through the crystal. Fur-

[1] Coblentz, Bul. Bur. Stds., 5, p. 376, 1908; Hyde, Jour. Franklin Institute, 170, p. 30, 1910.

[2] Stewart, PHYS. REV., 16, p. 123, 1903.

[3] Bul. But. Stds., 7, pp. 255–256, 1910.

[4] Bul. Bur. Stds., 7, p. 270, 1910.

[5] Bur. Stds., Sci. Paper No. 362, 1920.

[6] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

[7] American Physical Society, New York meeting.

[8] Coblentz and Kahler, Bur. Stds. Sci. Paper No. 338, 1919.

[9] Coblentz and Kahler, Jour. Wash. Acad. Sci., 9, p. 357, 1919.

thermore, this anomalous, photo-negative property is localized in one spot in the crystal.

These tests were most conveniently made at about − 100° C. Applying the potential to the sample in the direction so that the dark current (the electrical conductivity) was the smallest; then on increasing the applied voltage through various values from 10 to 70 volts, it was found that the photoelectric responses (the galvanometer deflections) were proportionately increased throughout the spectrum. On the other hand, reversing the potential so that the dark current was the greatest (the actual increase in conductivity was 4 times that observed for the opposite direction), it was found that for the region of the spectrum from $\lambda = 0.5\,\mu$ to $0.65\,\mu$ the electrical responses were photo-positive then became more and more photo-negative as the potential was increased from 10 to 70 volts, as previously observed.

A similar photo-negative reaction is obtainable when the current is flowing in the opposite direction through the crystal; but it requires the application of a far greater potential than that which obtains to produce equal dark currents in the two directions.

WASHINGTON,
Feb. 20, 1920.

ON THE X-RAY SPECTRUM OF TUNGSTEN.[1]

BY WILLIAM DUANE AND R. A. PATTERSON.

THIS paper describes accurate measurements of the critical absorption wave-lengths associated with the K and L series of x-rays characteristic of tungsten, and also of the wave-lengths of some of the L series emission lines.

The measurements were made by means of an ionization sepctrometer, similar to the one described in the PHYSICAL REVIEW, Dec., 1917; and the values obtained are compared with the best measurements of the same lines by the photographic method.

Using corresponding measurements of the K series made in collaboration with Dr. Stenström (to be presented in another paper), the relation between the two series published by Duane and Shimizu in the PHYSICAL REVIEW for July, 1919, has been tested, and found correct to within less than six one hundredths of one per cent.

The frequency interval of the "L series doublet" calculated by A. Sommerfeld from Bohr's theory of the mechanism of radiation on the assumption of elliptic orbits appears to be correct to within a fraction of one per cent.

ON THE K SERIES OF X-RAYS.[1]

BY WILLIAM DUANE AND WILHELM STENSTRÖM.

THE object of the research reported in this paper has been to measure the wave-lengths of the emission lines and that of the critical absorption in the K series of tungsten with a view to testing certain general laws and theoretical deductions.

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

In order to obtain as precise values of the wave-lengths as possible spectra of higher order than the first have been examined (the second, third, fourth and in one instance the fifth order).

Measurements of the electric current in the ionization chamber of an x-ray spectrometer were made at intervals of 30″ on the scale, giving the grazing angles of incidence of the x-rays striking the crystal. The tips of the peaks on the ionization grazing angle graph correspond to the lines in the characteristic x-ray spectrum, and it is possible to determine with certainty not only between which two settings of the crystal a tip lies, but also about how far from each it is. In other words the grazing angle of incidence can be estimated in each single set of good measurements to within about 7″ of arc. Since for the tungsten K series the grazing angles, θ, of the first order equal about $2° = 7200''$, this means that the errors of a satisfactory determination are less than $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{40}$, etc., per cent. in the spectra of the first, second, third, fourth, etc., orders respectively. The lantern slides shown illustrate these points.

The following table contains the weighted mean values of the wave-lengths, λ, in ångströms, and the corresponding wave-numbers.

Line.	α_2.	α_1.	β.	γ.	Abs.
λ.....	.21341 ± 3	.20860 ± 4	.18420 ± 3	.17901 ± 6	.17809 ± 7
$\frac{1}{\lambda}$.....	4.6858	4.7938	5.4290	5.5864	5.6151

Theoretically the α doublet should have the same wave-number interval as the L series doublet, the value of which, as given in a paper by Dr. Patterson and one of us, is .1081 ± .0002. This equals our value, .1073 ± .0015 to within the errors of measurement, which are larger in the K series than in the L, for the interval is a much smaller fraction of the wave-numbers in the former than the latter.

To test the law presented to this Society last year by Duane and Shimizu, namely, that the difference between two absorption frequencies equals an emission frequency we have taken two absorption wave-numbers belonging to the L series and given in the paper referred to above, namely, $La_1 = .8240$ and $La_2 = .9323$, subtracted them from the K series absorption wave-number and compared the differences with the K α emission wave-numbers, viz.,

	Authors.	Siegbahn.
$Ka - La_1 = 4.791 \pm .003$,	$K\alpha_1 = 4.7938 \pm .001$	$= 4.7898$,
$Ka - La_2 = 4.683 \pm .003$,	$K\alpha_2 = 4.6858 \pm .0007$	$= 4.6849$.

The absorption difference agrees with the emission wave-numbers to within the errors of measurement.

Dr. Stenström has observed three critical absorption wave-lengths associated with the M series of uranium and thorium. If we estimate the absorption wave-numbers in the M series of tungsten from the measurements he made

on the tungsten M series of emission lines, we find the approximate values

$$Ma_1 = .152, \qquad Ma_2 = .157, \qquad Ma_3 = .182.$$

Subtracting these from the K absorption wave-number and comparing with the $K\beta$ emission line, we find that one of the differences lies very close to the $K\beta$ wave-number, and that all three are so near it that their corresponding points on the wave-length graph fall within the peak representing the $K\beta$ line.

$$Ka - Ma_1 = 5.463,$$
$$Ka - Ma_2 = 5.459, \qquad K\beta = 5.429.$$
$$Ka - Ma_3 = 5.433,$$

This suggests a reason why Kossel's relation, namely, that the difference in frequency between the $K\beta$ and the $K\alpha$ lines should equal that of one of the lines in the L series, does not hold. If the $K\beta$ line is complex (as is the $K\alpha$ doublet) we must subtract the wave-number of a single line in the $K\alpha$ group from that of a single line in the $K\beta$ group to get the wave-numbers of a (possible) line in the L series.

With reference to the prevailing conception of electrons revolving in atomic orbits the electrons producing the $K\beta$ and the L lines may not fall from the same M orbits in sufficient numbers to produce perceptible lines.

THE EMISSION OF ELECTRONS FROM OXIDE-COATED FILAMENTS UNDER POSITIVE BOMBARDMENT.[1]

BY C. DAVISSON AND L. H. GERMER.

THE emission of electrons from solids under positive bombardment is a matter of some interest in connection with the study of thermionic filaments. In general, when an electron current is established from a glowing filament to an anode, a current of positive ions is established, at the same time, in the opposite direction. These either bombard the filament as positive ions, or recombine and bombard it (a part of them at least) as high-speed neutral molecules.

It is conceivable that under this bombardment the filament emits considerable numbers of secondary electrons which add to the true thermionic emission. From what is known of secondary emission from unheated metals under positive bombardment it is hardly to be expected that this contribution will be appreciable in the case of uncoated metallic filaments. Data have been lacking, however, to show that the same is true of oxide-coated filaments. It has even been suggested that oxide-coated filaments are not thermionically active, or only slightly so, and that their apparent thermionic activity is, in fact, due to their emitting copiously under bombardment.[2]

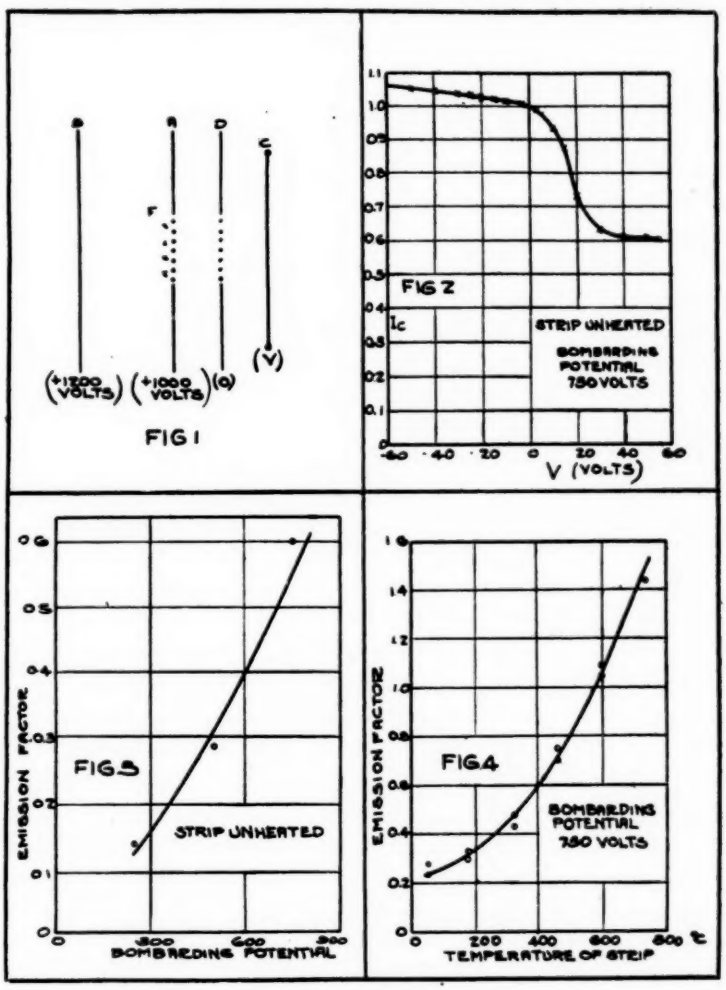
An investigation of this subject has led to the conclusion that in a reasonably

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

[2] I. Langmuir, PHYS. REV., II., 6, p. 484, Dec., 1913.

well pumped tube secondary emission from oxide-coated filaments is an altogether negligible factor.

A, *B* and *D* in the diagram of the apparatus, Fig. 1, are nickel plates. *A* and *D* contain small grid-covered windows as indicated, and *B* is solid. *F* is a tungsten filament, permanently connected at one point to *A*, and *C* is the oxide-coated filament under examination. It is of platinum foil coated with a mixture of barium and strontium oxides, and is somewhat broader and longer



Figs. 1–4.

than the window in *D*. If the potentials of the various parts are as indicated, it is clear that with *F* heated an electron current will be established from *F* to *B*; the gas in this region will be ionized, and the positive ions accelerated toward *A*. A part of these pass through the window in *A* and are further accelerated between *A* and *D*. Those that finally pass through *D* have, in

this case, velocities ranging from 1,000 to 1,200 volts equivalent. These are allowed to fall on C. If C is sufficiently positive to D any electrons emitted under this bombardment will be returned, while if it is negative to D they will escape. By comparing the currents to C under these two conditions one may determine the emission factor in the usual way.

At a pressure of 5×10^{-5} mm. the free path is great compared with the linear dimensions of the apparatus and currents to C are obtained of the order 10^{-7} amperes. So far measurements have been made only with the gas residual in the tube after baking and pumping.

The nature of the results is indicated by the curve in Fig. 2. The abscissæ are potentials of C relative to D, and the ordinates, the currents to C, the current to C for $V = 0$ being taken as unity. Beyond $V = +40$ the current is constant at 0.6. The number of positives reaching C is six tenths as great as the number of positives plus the number of secondary electrons. In other words, 6 positives cause the emission of 4 electrons. The emission factor is 2/3.

The curve, Fig. 3, represents the emission factor for the unheated filament as a function of the bombarding potential up to 750 volts.

Fig. 4 shows the emission factor at 750 volts bombarding potential as a function of the temperature of the strip. This increase with temperature is surprising. Similar tests that we have made on uncoated platinum indicate no variation in the emission factor up to 900° C.

Over the period of tests on the coated strip, during which it was often heated to high temperatures, its emission factor decreased under all conditions of bombardment, although during the same period its thermionic activity increased about thirtyfold. When the data for the curve, Fig. 4, were taken the factor for room temperature and 750 volts was about 0.2 instead of 2/3 as indicated by the curve, Fig. 1.

Observations cannot be made by this method at temperatures at which the thermionic emission from the strip is appreciable compared with the positive current. By another method observations have been made up to 900° C. and 1700 volts bombarding potential. The greatest secondary emission observed in any case was three electrons per positive.

These results in conjunction with data on ionization currents in well-pumped vacuum tubes indicate that for oxide-coated filaments the secondary emission under positive bombardment is, in general, less than one one-hundredth of one per cent. of the thermionic emission.

RESEARCH LABORATORIES OF THE AMERICAN TELEPHONE AND TELEGRAPH COMPANY AND WESTERN ELECTRIC COMPANY, INC.,
NEW YORK CITY,
February 18, 1920.

CRYSTAL ANALYSIS OF METALLIC OXIDES.[1]

BY WHEELER P. DAVEY AND E. O. HOFFMAN.

USING the powdered-crystal method of Hull, X-ray diffraction patterns have been taken of the following oxides, CaO, MgO, CdO, NiO and Al_2O_3. The first four of these all show the same type of structure, namely, a face-centered cube of the metal, interpenetrating a similar face-centered cube of oxygen, so that the combined lattices form a simple cube whose side is half that of the face-centered cube. This structure is, therefore, identical with that of NaCl. The length of the side of this simple cube is CaO, 2.37 A.; MgO, 2.09 A.; CdO, 2.36 A.; NiO, 2.10 A.

The Al_2O_3 was made by heating the hydroxide. The experimental pattern fits the calculated pattern for a rhombohedron, whose hexagonal axes each have a length of 4.86 A. and whose vertical axis has a length of 6.62 A. That is, except for the difference in axial ratio, the structure seems to be identical with that of calcite.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.

NOTE ON THE POSSIBLE NON-RADIALITY OF THE OBSERVED DEFLECTIONS OF LIGHT RAYS DURING THE SOLAR ECLIPSE OF MAY 29, 1919.[1]

BY LOUIS A. BAUER.

THE following investigation is based on the deflections of light rays observed, during the solar eclipse of May 29, 1919, by the British Astronomical Expedition, equipped and sent to Sobral, Brazil, under the direction of the Astronomer Royal of England, Sir Frank W. Dyson. The observer, Dr. A. C. D. Crommelin, used a 4-inch lens of 19-foot focus and an 8-inch cœlostat.

Let α be the total deflection of a light ray coming from a star, S, and passing through the sun's gravitational field and finally reaching the observer on the earth. Suppose α_0 be the value of α, if the ray grazed the sun's limb, and ρ, the radius vector or distance from the sun's center to the ray of light passing the sun. Then

$$\alpha = \alpha_0/\rho. \qquad (1)$$

According to the Newton-Maxwell law, $\alpha_0 = 0''.87$, and according to Einstein's, $\alpha_0 = 1''.74$. The observed effects appear to agree better with the Einstein law than with the Newton-Maxwell.

From Dr. Crommelin's observational results the resulting value of the deflection, α_0, at the sun's limb, as published by him, is $1''.98$,[2] thus agreeing with the Einstein predicted value, $1''.74$, within 14 per cent.

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

[2] See Nature, November 13, 1919, p. 281. The probable error as given by Dr. Crommelin is $0''.12$, whereas Dr. H. Spencer Jones of the Greenwich Observatory in his summary (Science Progress, January, 1920, p. 372) gives $0''.06$.

A further analysis of the observed light deflections shows that while the simple law (1) was followed to the greater extent, the effects in addition to varying inversely as the distance from the sun's center also apparently depended in some measure upon the heliographic co-latitude, θ, of the star. As a consequence the observed effects are not strictly radial, the departures from radiality occurring in a strikingly systematic manner, and not in the accidental manner that would be the case if the non-radial effects were attributable wholly to errors of observations. When such trigonometric functions are added to law (1) as would arise from forces similar to centrifugal ones, or such functions as express the variation with latitude of the gravitational force arising from an oblate ellipsoid of revolution (the earth, for example), the additional effects are largely accounted for. Resolving the observed actual deflections into two components, radial (along radius vector) and the other non-radial (perpendicular to radius vector), preliminary computations were made with the aid of the expanded law:

$$\alpha = \frac{\alpha_0}{\rho} + f(\rho, \quad \theta). \tag{2}$$

A value for α_0 resulted agreeing closely with the Einstein value of 1".74; instead of the value 1".98 stated above, the preliminary computations gave 1".77. (A slide was shown exhibiting the systematic character of the non-radial effects.)

It is, of course, impossible without further analysis to state at present just what portion of the observed effects may be accounted for by the various causes thus far suggested. Dr. Newall,[1] for example, is disposed to admit the possibility of the Newton-Maxwell effect, but prefers considering "that the balance of the observed deflection is attributable to optical refraction" (Refraction in the Solar Atmosphere). Further examination also shows that a measureable part of the observed light deflections may result from incomplete elimination of differential refraction effects in the earth's atmosphere. In this way both non-radial effects may be explained as well as the high value (1".98) of the deflection at the sun's limb.

If it should prove to be the case that the observed light deflections can be practically entirely accounted for by a combination of the causes stated in the previous paragraph, and Einstein's law of gravitation, the way is open to explain the results obtained by Dr. W. W. Campbell's eclipse expedition of June 8, 1918, at Goldendale, Washington. Using two 4-inch photographic objectives, photographs were taken of the sun and its surroundings, the exposures being 110 seconds and 50 stars to the ninth magnitude being recorded. He states his results as follows:[2] "Dr. Curtis divided his stars into inner and outer groups. The differential displacement between the two groups should have been 0".08 or 0".15, according to which of Einstein's hypotheses

[1] Monthly Notices, Roy. Astr. Soc., Vol. LXXX, No. 1, Nov. 1919.

[2] The Observatory, London, Vol. XLII., No. 542, August, 1919, pp. 298–300.

was adopted. The mean of the results came out at 0".05 and of the right sign. . . . It is my own opinion that Dr. Curtis's results preclude the larger Einstein effect, but not the smaller amount expected according to the original Einstein hypothesis."

It would be of great importance to know, of course, whether as the distance of a star from the sun greatly increases, the deflection of light will correspond more and more closely with that given by the Newton-Maxwell law. There is no possibility that the Einstein effect with increased distance will merge into the Newton-Maxwell effect, since theoretically the former should always be twice the latter.

The Einstein Deflection for High Speed Material Particles.[1]

By Leigh Page.

FOR the motion of a particle in the gravitational field produced by a body of comparatively much greater mass, Einstein's law of gravitation takes the form

$$\delta \int \sqrt{\left(1 + 2\frac{m}{r}\right)(dr^2 + r^2 d\theta^2) - \left(1 - 2\frac{m}{r}\right)c^2 dt^2} = 0,$$

where the $r\theta$ plane is that of the orbit, and m is the mass of the attracting body in astronomical units divided by the square of the velocity of light.

For a particle moving with a high velocity v, this equation leads to the relation

$$v^2 = c^2\left\{a^2\left(1 - 6\frac{m}{r}\right) + 2\frac{m}{r}\right\},$$

where a is the ratio of the velocity of the particle at infinity to that of light. If a is small, this reduces to the energy equation of the Newtonian theory. When, however, a is equal to $1/\sqrt{3}$, the velocity of the moving particle is unaffected by the gravitational field. In this case the acceleration of the particle (if any) is always normal to its path. For values of a greater than $1/\sqrt{3}$, the particle is actually slowed down as it approaches the attracting center, and speeded up as it recedes therefrom.

How a Reed Instrument is Played in Tune.[1]

By John B. Taylor.

REED and other wind instruments are often compared disadvantageously with the stringed instruments on the ground that they are of fixed pitch and, therefore, cannot equal the strings in their ability to make the fine alterations in pitch of a given note required for perfectly harmonious chords i n different keys.

The writer's experience and experiments show that the reed instruments

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

of the classical orchestra (oboe, bassoon, clarinet) permit, in the hands of a skilled player, sufficient pitch variation on any of the notes to meet the requirements of "just intonation," "equal temperament," etc.

While many musicians are familiar with the above fact, few seem to be aware of the means by which it is accomplished. There appears a prevailing opinion that it is accomplished by blowing harder or softer, or by "pinching" the reed more or less. While the force of blowing and "pinching" have some effect on the pitch, in practice the important feature which has been overlooked is that the vibrating air column does not end at the reed but extends through the reed into the mouth and throat cavities of the player.

It is by altering the size and shape of the vocal cavities that the nice adjustment of the pitch is made to meet the requirements of the art.

Demonstrations will be made with a bassoon and other reed instruments showing the influence of the air cavity back of the reed, as well as that of the air column in the instrument, on the rate of vibration of the reed.

GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.,
February, 1920.

FURTHER DEVELOPMENTS RELATIVE TO THE NEW SINGING TUBE.[1]

BY CHAS. T. KNIPP.

(*a*) *Mode of Vibration.*—The true mode of vibration of the air column within this singing tube, when it is sounding its fundamental, was brought to light recently by C. J. Lapp while measuring photographs of the wave-length of the tone emitted. These measurements show that the *total length of the vibrating column is one quarter of a wave-length.* This means that the point of application of the heat energy is not a loop as I had supposed. This view of the mode of vibration is further substantiated by observations made previously by two of my students, in which the points of resonance in a resonating column were obtained and these lengths compared with the lengths of the various branches of the singing tube. The true relation, at the time, was overlooked because it was supposed that the point of application of the heat must necessarily be a loop.

A formula has been established which gives the length of the equivalent resonant column for any position of the water piston. The constants of the equation were obtained from the dimensions of the tube when sounding its fundamental and at the same time emitting a pure note,—a condition which the later photographs of the wave-form indicate is quite possible of obtaining.

This newly established relation, that the total length of the vibrating column is one quarter of the wave-length, explains the low tone emitted by this form of singing tube.

(*b*) *Note on Operating Conditions.*—It has been shown experimentally that this form of tube operates with the least expenditure of heat energy when its

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

point of application is about one third of a quarter wave-length from the inner closed end (this in the L-form of tube being the distance down to the water piston, and in the straight-form being the length of the inner test tube). Straight-forms have been constructed that will operate on the heat from the burning of one match. This suggests the following: Will the tube emit a tone by the application of liquid air only? To test this a straight-form tube was used. It was arranged to surround the entire tube with liquid air except the tip that is ordinarily heated. To begin the tube was at room temperature. On filling the jacket with liquid air *the tube at once began singing* and increased in intensity (the pitch falling slightly) until the jacket was filled. In a few minutes the intensity began to decrease, due to the tip being cooled by conduction through the glass and air within. The application of a lighted match to the tip made the tube fairly roar.

To make this experiment work to the best advantage the tip should be blown from exceedingly thin glass. It may be added that the tube will not sound when the inverse operation is performed, *i.e.*, when the tip is dipped in liquid air.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
February, 1920.

THE DISTRIBUTION OF ENERGY ABOUT A POINT SOURCE.[1]

BY PAUL R. HEYL.

IT is usually stated that the intensity of the energy about a point source varies, in two dimensions, inversely as the distance from the source, and, in three dimensions, inversely as the square of the distance. It is here shown that in the case of stationary waves with circular or spherical symmetry this statement is only approximate.

In the case of the transverse vibrations of an elastic membrane the outline of a free wave is that of a Bessel's Function, the maxima and minima of the successive loops being displaced slightly from the center of the loop toward the origin, this displacement approaching zero as the distance from the origin increases. The energy at such points is a little less than that called for by the law of the inverse radius, the deficiency being about two and one third per cent. at the first loop, and about one twentieth of one per cent. at the sixth loop. At the center points of the loops a similar law holds.

In three dimensions, such as sound in air, a similar condition prevails. The outline of a free wave is given by the function $(\sin r)/r$ whose maxima and minima are displaced similarly to those of a Bessel's Function. However, in the case of the central points of the function $(\sin r)/r$ the energy is proportional to the inverse square of the radius.

Splitting this stationary wave-system up into its two oppositely moving components, each is seen to possess the same characteristic displacement of

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

the maxima and minima. As the conservation of energy demands, the time integral of any loop at a given point is proportional to $1/r^2$.

The interest of this peculiarity lies in its suggestiveness. The law of classical mechanics are, as usually expressed, apt to be simple and well rounded, and therefore sometimes fail to fit perfectly the observed facts. An instance of this is the Newtonian Law of gravitation. To explain the observed discrepancy in the motion of the perihelion of Mercury a slight departure from this law is necessary; a departure which is perceptible at short distances, but disappears at great ones. Such a modification of the Newtonian law has, in fact, been lately given by the theory of relativity, and (to a partial extent) by Silberstein on the basis of the variable mass of a moving body. Though the case described in the present paper has no apparent connection with gravitation, it is interesting as suggesting further possibilities in the critical study of the older mechanics.

THE COMMERCIAL RESEARCH COMPANY,
NEW YORK CITY.

THE SELECTIVE REFLECTION OF HEAT WAVES BY LINEAR RESONATORS.[1]

BY E. C. WENTE.

A LARGE number of experiments have been carried out on the reflection of electric waves by groups of linear resonators arranged in parallel rows and columns over a plane surface. These experiments show that the amount of energy reflected is a maximum when the wave-length is approximately equal to twice the resonator length. This wave-length corresponds to the free period of the resonator. The first experiments of a similar nature with the shorter electromagnetic waves obtained by purely thermal means were undertaken by Rubens and Nichols[2] in 1897 while working with the residual rays of fluorite, which have a mean wave-length of 24 μ. Because of the difficulty of making resonators of microscopic dimensions only a few reflecting plates were constructed. However, it was found that more energy was reflected when the resonator length was an even multiple than when it was an odd multiple of the quarter wave-length. Wood[3] has since carried out some resonance experiments on long heat waves in a somewhat different way, but was unable to detect any resonance effects. The experiments here described were undertaken for the purpose of determining whether the results that have been found for electric waves also obtain in the case of heat waves and linear resonators of similar type.

In order to get radiation of long wave-length and of sufficient purity, the radiation obtained from a Welsbach burner with the quartz lens focal isolation method was reflected[4] in succession from two surfaces of potassium iodide,

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

[2] PHYS. REV., 5, 164 (1897).

[3] Phil. Mag., 25, 440 (1913).

[4] Rubens & Wood, Phil. Mag., 21, 249 (1911).

which reflects selectively waves having a length of about 95 μ. The intensity of the radiation was measured by means of a specially designed sensitive vacuum thermo-couple connected to an astatic galvanometer. The purity and wave-length of the resultant radiation was determined by means of a quartz plate interferometer of the Fabry and Perot type.[1] The measurements showed a mean wave-length of 96 μ. The radiation was rendered plane

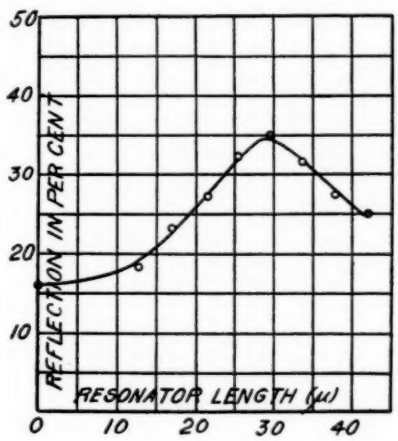


Fig. 1.

polarized by placing in the path of the rays a wire grating, which had a constant of 50 μ.[2]

The linear resonators were prepared by ruling chemically silvered glass surfaces with a diamond on a dividing engine. In this way a series of plates were made on which the resonators were distributed uniformly in parallel rows and columns. On each of the plates the axial separation of the resonators was equal to 45 μ, and the endwise separation, 4 μ. The length of the resonators on the difference plates varied in 4.2 μ steps from 13 to 43 μ, the width being 8 μ in all cases.

The percentage reflection of each of the plates for the 96 μ radiation was measured. The results are shown graphically in the figure. The curve shows that there is a distinct maximum of reflection for a resonator length equal to one third of a wave-length. This value is almost exactly the same as has been found in the case of electric waves emitted by a Righi oscillator and resonators made of tin foil when attached to a glass plate.

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[1] Rubens & Hollnagel, Sitzb. der Preuss. Akad., Jan. 20 (1910).
[2] Du Bois & Rubens, Ber der D. Phys. Gesell., 9, 43 (1911).

ON ELECTROSTATIC TRANSFORMERS AND COUPLING COEFFICIENTS.[1]

BY F. C. BLAKE.

IT is shown that the capacity of an air condenser for high frequency currents can be expressed by the formula $C = C_{11}/(1 - k^2)$ where C_{11} is the self-capacitance of either of the two equal conductors constituting the condenser and k is the coefficient of coupling, *i.e.*, it is the ratio of the square root of the product of the two self-capacitances divided by the mutual capacitance between the two conductors. These definitions of self and mutual capacitances arise as the reciprocals of self and mutual elastances which are proved to be in every way the exact analogues of self and mutual inductances. It is further shown that the coefficient of elastic coupling is equal to the mutual elastance divided by the square root of the two self-elastances and that the coefficient of electrostatic coupling between two circuits each of which contains a condenser with a third condenser in the branch common to the two circuits is identical with this coefficient of elastic coupling. Ordinary condensers are thus shown to be electrostatic transformers whose coefficient of elastic coupling is unity.

Important physical consequences in connection with triode valves in radio work follow as a result of the considerations of this paper.

INFRA-RED SIGNALING.[2]

BY W. W. COBLENTZ.

SURPRISING as it may appear, it can be said without exaggeration that in the secret transmission of intelligence by means of invisible, thermal radiations, radiometry attained its greatest triumphs, and one of its most far-reaching applications.

In the matter of signaling by thermal radiometric apparatus, this Bureau performed two distinct functions, viz., (1) it acted in an advisory capacity, and, (2) it carried on investigations, having in view the improvement of the sensitivity of radiometric receivers and transmission screens. Incidentally the Bureau was prepared to test the relative merits of thermal radiation signaling devices; also as instrument maker it loaned thermopiles, etc., for special tests in battleship and submarine detection. Moreover, in view of the facts just enumerated, some attention was given to methods of detecting such signaling devices.

Among the inquiries received, the subject most frequently presented, related to non-selective thermal-radiodynamic devices for signaling, and for detecting submarines, or battleships in a fog, by the heat radiated from the smokestacks, etc. Most of the inquirers did not seem to realize that water vapor is the most opaque substance known, for radiations of wave-lengths such

[1] Abstract of a paper presented at the New York meeting of the American Physical Society, February 28, 1920.

[2] Abstract of a paper presented at the Washington Meeeting of the American Physical Society, April 26, 1919.

as would be emitted by a smokestack, and that consequently, during a fog, when this sort of signaling device would be most needed, the radiations to be detected would be completely absorbed within a very short distance from the source.

Many of the questions submitted were so impractical that it was easier to advise what *not* to do, than to suggest improvements in the schemes proposed.

Several writers (Rühmer in Germany and Miessner in U. S. A.) have written books discussing secret signaling devises. Selenium was practically the only substance then known (1913) which was photoelectrically sensitive, and which was discussed as a means of signaling.

This Bureau (Div. IV–5) became actively interested in this subject in 1915 but no extensive researches were begun until early in 1917, when a search was made for a substance which had a marked electric sensitivity for infra-red rays. Aside from tests made with receivers of gold-leaf, blackened on one side with soot, no attempt was made at improvement of the non-selective radiometers, of the blackened bolometer receiver type, in view of their sensitivity to low temperature radiation and hence unsteadiness when exposed as they would be when searching at night for objects such as floating submarine, a man outside of a trench, or an air plane in the unilluminated sky.

This conclusion that a non-selective radiometer is unsuitable as a receiver in secret signaling was amply verified by others in actual field tests. Men wearing heavy coats and having their faces covered were difficult to detect; similarly with boats having the shore line for a background. Clouds of water vapor also emit radiation which interferes with the observations.

It was found that some samples of molybdenite are photoelectrically sensitive for infra-red rays. This property as well as its quickness of response renders this substance superior to selenium as a radiophonic signaling device, employing a pulsating electric current which is produced by rotating a sectored disk in front of the receiver, so as to interrupt the incident radiations.

The radiophonic signaling apparatus used in our tests consisted of a crystal of molybdenite and a battery of dry cells, giving 40 to 60 volts, which were connected directly to the input terminals of a 3-stage amplifier. A concave silver-on-glass mirror, 16 cm. in diameter and 50 cm. focal length was used to concentrate the radiations upon the molybdenite receiver. For a field outfit the mirror, sectored disk, (operated on 6-volt battery) and molybdenite receiver were mounted upon a camera tripod.

The radiation sensitivity of molybdenite sample No. 1 was compared with the gold-leaf receiver, by exposing it directly to a standard of radiation, using minimum audibility as a criterion for making the comparison. The results showed that molybdenite sample No. 1 was 150 to 200 times as sensitive as the gold-leaf radiophone.

Field Tests.

For making our tests,[1] the most convenient signaling stations were the roof of this Bureau's electrical building and the clock tower of the U. S. Soldiers

[1] Made with the assistance of Mr. H. Kahler and Mr. M. B. Long.

Home—a distance of 3 miles. The intervening space was partly occupied by city houses and streets, with dust, etc.

A test of the field outfit was made, the source of radiation being a (250-watt) 300 c.p. tungsten lamp in a 28 cm. searchlight reflector, at a distance of 3 miles. The signals transmitted by the absorption screens were loud and easily perceived, in spite of a high wind and other disturbances.

Using an automobile headlight, consisting of a 20 c.p. (say 20 watts) tungsten lamp in a 15 cm. metal reflector, the signal through the absorption screen was not always heard, showing that this was about the limit of audibility. When this field outfit was sighted upon a street lamp (tungsten filament in a clear bulb; 80 c.p.) at a distance of one half mile, a clear audible note was heard in the telephone, when no absorption glass intervened; but the signal was uncertain when using only infra-red rays.

Interesting results were obtained using the full moon as a source of radiation. The infra-red lunar radiations caused a loud sound in the telephone receiver, although the intensity was only 3×10^{-7} gr. cal. per second.

Comparison of Signaling Methods.

The method of producing a pulsating current in the telephone by interrupting the light which is incident upon the molybdenite, by interposing a rotating sectored wheel, is inefficient in view of the fact that (1) only one half of the incident light is utilized and (2) the exposure-time is only about 1/500 of a second, which is not sufficient to permit great change in the photoelectrical conductivity. A signaling system which can utilize a longer exposure of the receiver to the incident light will obtain a greater change in the electrical conductivity.

When using the sectored wheel radiophone, the signal is recognized by the musical note emitted by the telephone receiver, the loudness of which is determined by the intensity of the incident radiations. The signal could be recognized also by a change in pitch of the sound in the telephone receiver provided apparatus can be devised to function by changing the pitch of the sound. This change in pitch was frequently observed in the amplifier used in these experiments.

The change-in-pitch method of signaling which is not original with the writer, seemed so attractive that apparatus was devised in order to obtain a test of its efficiency as compared with the sectored disk radiophone, using, in both devices, the molybdenite for the receiver of the thermal radiations which were used in transmitting the signal. The result of this test showed that owing to a high resistance which had to be connected in series with the single receiver of molybdenite but which did not function photoelectrically when exposed to light, the method of signaling by change in pitch was not so sensitive as the rotating sectored disk radiophone. Using several molybdenite receivers, joined in series, so as to obtain the required high resistance, when exposed to light, the sensitivity of this device was improved.

A photoelectric cell of the gas-ionic type, *e.g.*, the potassium hydride photoelectric cell, is well adapted for use with this change-in-pitch method of signaling. The high resistance of the cell is used as ballast to the photoelectric cell. This combination was found to be the most sensitive of the radiophonic type of receivers yet tested. The applied voltage can be adjusted so that the telephone receiver emits a sound only when the photoelectric cell is exposed to light.

In conclusion it is pertinent to add that through many contributory circumstances the accomplishments of this Bureau fell far short of what could have been done if full advantage had been taken of the facilities that were available.

On the other hand, some of the real accomplishments of this country had better be locked in the military archives for future reference—locked there but not buried there.

BUREAU OF STANDARDS,
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